A STUDY ON THE JOINT PROBABILITY DISTRIBUTION OF WIND AND WAVE

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A model for joint probability distribution of wind and wave is proposed and relations between wind speed, wave height and wave period at deep water near the Choshi offshore wind demonstration site are derived by using 10 years simulation data, which are obtained from mesoscale and third generation wave models. Mean values, standard deviations, probability distributions of wave height and wave period are expressed as functions of wind speeds and modeled by combined models of wind wave and swell, linear functions and log-normal distributions, respectively. The correlation coefficient between wave height and wave period is also modeled as a function of wind and expressed by a hyperbolic tangent function. Monte Carlo simulation is conducted, and wind speeds, wave heights and wave periods are generated based on proposed models. Predicted joint probability distributions show satisfactory agreement with observations.

Keywords: Joint probability distribution of wind and wave, Correlation coefficient between wave height and wave period

INTRODUCTION

Long-term joint probability distribution of wind speed, wave height and period is required as external environment conditions for the fatigue load analysis in IEC 61400-3 [1], which is international standard for the design of offshore wind turbine. However, the joint probability distribution has not been defined as formula but calculated by using scatter plot which directly obtained from observations and/or simulations. Problem of current method is that the distribution might be unstable when it calculated from short-term observations.

Mean relations of wind speed and significant wave height/period were modeled by Sverdrup, Munk[2] and Bretschneider[3], so-called SMB method. Since, the models aim to predict wind-wave, these are not accurate in swell dominant wave, which occurs under low wind speed. To overcome this problem, Ishihara et al. [4] proposed combined models of wind-wave and swell, and these show good agreement with mean relations of observed wind speed and significant wave height/period. For the modelling of these joint probability distribution, in addition to these conventional models, standard deviations, probability distributions and correlation of wave height and period need to be modelled.

In this study, 10 years weather and wave simulation’s results are analyzed and a joint probability distribution model of wind speed, significant wave height and period is proposed, in which mean values, standard deviations, probability distributions of wave height/period and correlation coefficients of these are modelled as functions wind speeds. Then, Monte-Carlo simulation of proposing model is carried out and the result is compared with observation at Choshi offshore wind energy test site.

SIMULATION AND MEASUREMENT

Weather and wave simulation

In this study, a joint probability distribution model was constructed by analyzing winds and waves obtained from 10 years weather and wave simulation. For the weather and wave simulation, mesoscale model WRF (Weather Research and Forecasting model) and third generation wave model WW3 (Wave Watch III) were used, respectively. Simulations were carried out at Choshi offshore site, which offshore wind energy demonstration project has been conducted and both wind and wave have been observed since Feb. 2013. The site is located 3.1km offshore, at which water depth is 11.9m. Since wave at the site may be affected diffraction, reflection, deformation, and so on, simulation output values at 35.12°N latitude and 140.82°E longitude where water depth of 470m in model grid were used for the modeling. The locations of measurement site and simulation output grid are shown in Fig. 1.

Wind speeds at 10m height, significant wave heights and periods were used for the modeling. Where, significant wave height, $H_{1/3}$, and period $T_{1/3}$, are defined as the mean of one-third highest waves and the mean period of these, which are obtained by zero-crossing analysis, in one record. For observations, 20-minutes averaging time are commonly used in Japan and also used in this study. On the other hand, wave model predicts energy spectrum, $E(f, \theta)$, as functions of frequency, $f$, and direction, $\theta$. Due to this, following formulas

![Fig. 1 Locations of the simulation output and observation site.](image-url)
are used to convert from a spectrum to significant wave height and period:

$$H_{1/3} = 0.956H_{m0} = 0.956 \times 4 \sqrt{m_0}$$  \hspace{1cm} (1)

$$T_{1/3} = T_{m-1/3} = m_{-1}/m_0$$  \hspace{1cm} (2)

where, $m_n$ is the $n$-th order moment of $E(f, \theta)$ shown in following formula:

$$m_n = \iint f^n E(f, \theta) df d\theta$$  \hspace{1cm} (3)

For the modeling, predicted wind speeds, significant wave heights and periods during only sea breeze are used. This is because that characteristics of the relation between wind and wave are completely different by winds from land or sea. An advantage to make model by using only sea breezes is that it is easy to understand these relations. Moreover, the model obtained from these may be conservative when it is used for fatigue load analysis. For this reason, the model is constructed by using only sea breezes in present paper. In this study, wind directions from 78.25° to 213.75° were defined as sea breeze.

10 years simulations from 2000 to 2009 were carried out and every 20 minutes output values were used for the modelling. Note that a couple of these with predicted wind speed of more than 22m/s, at which the number of data in 1m/s wind speed bin become less than 30, were not used because statistic values would be unstable. Other details about weather and wave simulations see Tanemoto and Ishihara[5].

**Measurement data**

Observed wind and wave data from Feb. 2013 to Jan. 2014 at Choshi offshore site were used for validation of the joint probability distribution model. At the site, wind speed and direction has not been observed at the height of 10m. For this reason, wind speeds at the height were calculated by using power low, in which every 10-minutes wind shear exponents, $\alpha$, were identified by average wind speed observed by 3 cup anemometer mounted on 60m and 80m. For the classification of land/sea breeze, wind directions observed by vine on 80m were used.

As wave observations, significant wave height and wave period of 20-minutes statistic values were used. These are observed by from super-sonic wavemeter and calculated by zero up-cross analysis. Since wave statistics are obtained every 20-minutes, 10-minutes average wind speeds and directions on these time were used in this study. Other details about observations see Fukumoto et al.[6]

**A JOINT PROBABILITY DISTRIBUTION MODEL FOR DEEP WATER SITE**

Proposing formula for joint probability distribution model of wind and wave is summarized as table 1. The model is composed by means values, standard deviations, probability distributions of significant wave height and period and these correlation coefficient. All of these are defined as functions of wind speeds. Statistic values for each 1m/s wind speed bin were calculated by using weather and wave simulation result. Then appropriate functions was selected and coefficients were identified by these values. In following sections, characteristics of these formulas are explained.

**Mean and standard deviation models**

As formulas for the mean relations between wind speeds and significant wave height/period, combined models of wind-wave and swell proposed by Ishihara et al.[4] are used. $\mu_{H_{1/3},W}$ and $\mu_{T_{1/3},W}$ in Table 1 are mean values of wind-wave components for significant wave height and period. $F$ and $g$ are fetch and gravity acceleration. In this study, 235,000m defined by Ishihara et al. [4] and 9.81m/s² are used, respectively. $\mu_{H_{1/3},S}$ and $\mu_{T_{1/3},S}$ are mean values of swell component for significant wave height and period. These values are defined as mean values for wind speed bin of 0-1m/s, in this study. For the standard deviation models, linear functions are used and these constants, $a$, $b$, $c$, and $d$, are identified by using least-squares

<table>
<thead>
<tr>
<th>Mean values</th>
<th>$H_{m0} = \sqrt{H_{m0}^2 + \mu_{m0}^2}$</th>
<th>$\mu_{m0} = \frac{0.30U_0}{g} \left(1 + 0.004 \left(\frac{gF}{U_0^2}\right)^{0.13}\right)$, $\mu_{m0} = 1.24$; where, $g = 9.81$ m/s², $F = 235,000$ m</th>
<th>$H_{m0} = \frac{0.74\alpha_{U0}}{g} \left(1 + 0.008 \left(\frac{gF}{U_0^2}\right)^{0.33}\right)$, $\mu_{m0} = 7.75$; where, $g = 9.81$ m/s², $F = 235,000$ m</th>
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<tbody>
<tr>
<td>Standard deviations</td>
<td>$\sigma_{n0} = aU_0 + b$; where, $a = 0.0323$s, $b = 0.318$m</td>
<td>$\sigma_{n0} = cU_0 + d$; where, $c = -0.0210$m³, $d = 1.56$m</td>
<td></td>
</tr>
<tr>
<td>Probability distribution Functions</td>
<td>$f(H_{1/3}) = 1 + \frac{1}{2} \ln \left(\frac{H_{1/3}}{\lambda_{0/3}}\right)$, $\lambda_{0/3} = \ln \mu_{n0} - \frac{1}{2} \sigma_{n0}^2$, $\sigma_{n0}^2 = \ln \left(1 + \frac{\sigma_{n0}^2}{\mu_{n0}}\right)$</td>
<td>$f(T_{1/3}) = 1 + \frac{1}{2} \ln \left(\frac{T_{1/3}}{\lambda_{0/3}}\right)$, $\lambda_{0/3} = \ln \mu_{n0} - \frac{1}{2} \sigma_{n0}^2$, $\sigma_{n0}^2 = \ln \left(1 + \frac{\sigma_{n0}^2}{\mu_{n0}}\right)$</td>
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<tr>
<td>Correlation coefficient</td>
<td>$R_{H_{1/3},T_{1/3}} = \tanh(aU_0 + \beta)$; where, $a = 0.0484$m/s, $\beta = 0.643$</td>
<td></td>
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Mean and standard deviation of significant wave height as functions of wind speeds are shown in Fig. 2. The wind-wave model underestimates wave height in the region of low wind speeds where wave heights are determined by swell. On the other hand, by using combined model, these underestimations are improved and the models show good agreement with values obtained from numerical simulations. Standard deviations of wave heights for each wind speed bin also get higher as wind speeds become stronger, and these relation can be approximated by using linear function.

Fig. 3 shows mean and standard deviation of significant wave period as functions of wind speeds. In general, the frequency distributions for annual wave height can be approximated by Reighley distribution[7]. However, in this study, models for each wind speed bin is needed. In such a case, concentration of frequency distribution which depend on the strength of wind speed would be occurred. For example, in fig. 4-(a), the frequency of occurrence for low wave height is quite low under the wind speed bin for 12-13m/s. As another example, low wave period ($T_{1/3} < 5s$) is hardly occurred through both wind speed bin because of the appearance of swell. Due to this reason, log-normal distribution which has two parameters was used for these modelling.

Correlation model of wave height and wave period

Correlation coefficient between significant wave height and period as a function of wind speed are modelled by using hyperbolic tangent function. Constant values, $a$ and $b$, are identified by least squares method. Fig. 5 shows correlation

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig5.png}
\caption{Cumulus frequency distributions for significant wave heights and periods for 4-5 and 12-13m/s wind speed bin.}
\end{figure}
coefficients predicted by numerical simulations and modelled function. The correlation coefficients for low wind speed range take lower value than these for high speed range because of the effect of swell. Under high wind speed, these correlation become high due to the domination of wind-wave. These characteristics can be well approximated by using proposing formula.

VALIDATION OF THE JOINT PROBABILITY DISTRIBUTION MODEL

By using proposed joint probability distribution model in the chapter above, Monte-Carlo simulation were carried out and the result was validated by using observations at Choshi offshore site. Weibull distribution was used for the frequency distribution of annual wind speed, for which shape parameter, \( k = 1.79 \), and scale parameter, \( C = 6.18 \), were used, respectively. At first, a wind speed which satisfies the Weibull distribution was generated. Then, a couple of significant wave height and period was calculated from the wind speed by using formulas in table 1. Where, since the water depth of the site is 11.9m, shallow water deformation and braking wave effect were taken into account to generated wave height. In this study, Goda[7] formula was used as these model, in which diffraction and reflection coefficients were set as 1. According to the process in fig 5, wind speeds, significant wave heights and periods which correspond to every 20-minutes of 100years were generated.

Joint probability distributions obtained from observations and Monte-Carlo simulation are shown in fig. 6. It is found that frequency of occurrences take values of 0 at 10-15m/s of wind speed and 4m of significant wave height in fig. 6-(a), and 10s significant wave period in fig. 6-(b). These unnatural distribution is due to missing or insufficiency of the number of data. The frequencies obtained from Monte-Carlo simulation shows more natural and wider distribution, and these also success to predict the high frequency parts seen in distributions obtained from observations. Black lines in fig. 6-(c) and -(f) show the relation between significant wave height and period under the “well developed wind-wave”. In general, wave heights higher than the line are hardly observed under the period shown in x axis. The relation can be predicted by using proposed model.

CONCLUSION

In this study a joint probability distribution is proposed and the following conclusions were obtained:

1. 10-years numerical weather and wave simulation’s results were analyzed and a joint probability distribution of wind and wave were proposed.

2. 100-years wind speeds, significant wave heights and periods were predicted by using Monte Carlo simulation based on proposing model. Predicted joint probability distribution were show satisfactory agreement with observation.

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