Seismic Load Evaluation of Wind Turbine Support Structures with Consideration of Uncertainty in Response Spectrum and Higher Modes

T. Ishihara\textsuperscript{a}, G. Takamoto\textsuperscript{b}, M. W. Sarwar\textsuperscript{c}

\textsuperscript{a}ishihara@bridge.t.u-tokyo.ac.jp \textsuperscript{b}takamoto@bridge.t.u-tokyo.ac.jp \textsuperscript{c}sarwar@bridge.t.u-tokyo.ac.jp

Department of Civil Engineering, The University of Tokyo.
7-3-1 Hongo, Bunkyo-ku, Tokyo, 113-8656 JAPAN

Summary

Rapid development of wind energy in seismically active regions like Japan requires evaluations of design seismic load for support structures to ensure structural integrity. This paper presents a rational approach to determine design response spectrum for structures with low damping ratio such as wind turbine towers. A modified damping correction factor for the design response spectrum of wind turbine towers is proposed to incorporate uncertainty in seismic loads. Proposed correction coefficient of the design response spectrum is defined as a function of percentile quantile of the seismic response distribution, which is determined by code calibration method against the current seismic design loads. This study also presents a simplified formula for the seismic load evaluation of wind turbine towers based on response spectrum analysis. First three tower modes are found to have significant contributions to the seismic load of wind turbine towers and expected maximum seismic load is obtained by combining three modal responses using SRSS method. Expressions for estimation of first three mode shapes and natural periods are proposed to facilitate the routine design implementation. Finally the accuracy of proposed design response spectrum and seismic load formula is evaluated through comparison of results with time history analysis for wind turbines of capacities from 400kW to 2MW.

1. Introduction

Estimation of seismic response of wind turbines becomes of great importance when wind farms are designed and developed in seismically active regions. Being a seismically active region, Japan has strict regulations for designing and assessing the safety of wind farms. This require use of design formula based on response spectrum method and time domain analysis for low and high structures respectively. Current practice of estimating design loads based on the response spectrum method encounter two problems when applied to the wind turbine support structures\textsuperscript{1}. These support structures are extremely low damped and experience a wide range of frequencies when subjected to seismic activities. Response spectrum for such low damped structures show excessive fluctuation and such uncertainty in response spectrum can not be captured by existing models of the damping correction factors defined in Eurocode\textsuperscript{2} and BSL\textsuperscript{3}. In addition, use of the simplified SDOF model suggested by IEC\textsuperscript{4} results in linear vertical load profiles. However, vertical distribution of the seismic loads is found to be largely affected by the higher modes\textsuperscript{1} of wind turbines. Therefore simplified but accurate analysis method to estimate design load profiles is desired.

In this research, a modified damping correction factor that accounts for uncertainty in response spectrum, and modal participation functions encompassing complex vertical distribution of seismic loads are proposed. The accuracy and reliability of the proposed method for evaluation of seismic design loads is examined against time history analysis and current design codes.

2. Seismic Load Estimation

Generally the seismic design loads are estimated by two methods, time history analysis and the response spectrum method. In time history analysis, equation of motion is solved for different earthquake waves to obtain loads, shear and bending moments, acting on the wind turbine support...
structure. This method has inherent advantage of including the effect of higher modes and geometrical non-linearity on the structural response when structure is subjected to dynamic excitation. However, this method is relatively time consuming and requires certain number of seismic waves to account for variability of the analysis results. However, response spectrum method requires only natural period, mode shape and mass distribution of the structure to calculate maximum seismic loads.

This section describes basic assumptions used in the seismic load analysis, introduces equation of motion for time history analysis, response spectrum analysis method along with acceleration spectrum and, finally details of the wind turbine models used in this study are discussed.

2.1 Analysis assumptions

Basic assumptions for estimation of seismic loads acting on the wind turbine support structures are listed below:

a. Fixed support model is used for eigen value and dynamic analysis of the wind turbine support structure. Seismic waves at the tower base level are used.

b. The wind turbine tower is modeled as multi-degree of freedom system. Rotor and nacelle masses are modeled as a lumped mass situated at hub height.

c. To determine seismic loads under operating conditions, seismic load under parked conditions is combined with the operational wind loads.

d. Additional loads caused by geometrical non-linearity such as P-∆ effect and twisting at top of the tower are considered separately.

2.2 Time History Analysis

In time history analysis, seismic loads acting on the wind turbine are estimated by introducing the synthetic earthquake waves at the tower base. These waves are generated to closely match the target spectrum defined by equation (4). In this study, wind turbine tower is modeled as a multi-degree of freedom (MDOF) system, as shown in Figure 1, where mass of tower members are lumped at nodes and beam element model is used to model stiffness and damping of the respective members. Under parked conditions, the equation of motion for MDOF system subjected to ground acceleration $\ddot{x}_g$ is:

$$ [m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} = -[m]\{e\}\ddot{x}_g $$

where $\{\ddot{x}\}, \{\dot{x}\}, \{x\}$ are the acceleration, velocity and displacement vectors respectively. $[m], [c]$ and $[k]$ are the mass, damping and stiffness matrices. $\{e\}$ is a unit vector.

2.3 Response Spectrum Method

Response spectrum method is based on acceleration response spectrum of SDOF system that is used to determine maximum response of MDOF system. Following is the equation of motion for jth mode of a MDOF system,

$$ \ddot{q}_j + 2\zeta_j \omega_n q_j + \omega_n^2 q_j = -\gamma_j \ddot{x}_g $$

where $\omega_n$ is the natural angular frequency. $\zeta_j$ is the damping ratio. $\gamma_j$ is the modal participation factor. Maximum force for each mode depends upon the modal participation factor, mode shapes and acceleration of response spectrum corresponding to natural period of respective modes. For example, the maximum shear force corresponding to j-th mode of the i-th node can be estimated as follows:

$$ F_{ij} = \gamma_j X_j S_a(T_j, \zeta)m_i $$

- 2 -
where $X_{ij}$ is the $j$-th mode shape, $S_x(T, \zeta)$ is the amplitude of acceleration response spectrum corresponding to the natural period $T$ and damping ratio $\zeta$. To estimate maximum design loads, first load for each mode of the MDOF system is determined using equation (3) and then loads for all modes are combined. As response spectrum method uses dynamic characteristics such as mode shapes and natural period of the structure, a prior knowledge of these characteristics is required to estimate the seismic loads. Therefore, to facilitate the load estimation procedure, this study proposes a model to estimate these characteristics without eigen value analysis.

2.4 Acceleration Response Spectrum

The design acceleration response spectrum ($S_a$) used to determine seismic loads is defined as \[ S_a(T, \zeta) = \begin{cases} a_0 \cdot S \cdot \left[1 + \frac{T}{T_B} \left(\beta_0 \cdot F_\zeta - 1\right)\right] & (0 \leq T < T_B) \\ a_0 \cdot S \cdot F_\zeta \cdot \beta_0 & (T_B \leq T \leq T_C) \\ a_0 \cdot S \cdot F_\zeta \cdot \beta_0 \cdot \left(\frac{T_C}{T}\right) & (T_C < T) \end{cases} \] \[ F_\zeta(\zeta) = \left(\frac{\zeta}{2 + 100\zeta^2}\right)^{\alpha} \] (4)

Where $a_0$ is the peak ground acceleration at the engineering bed rock for a given return period, $S$ is the soil amplification factor, $F_\zeta$ is the damping correction factor and $\beta_0$ is acceleration response magnification ratio for the region where acceleration response becomes constant. $T_B$ and $T_C$ defines the range of constant spectral acceleration. Parameters to define a design acceleration response spectrum with a return period of 500 years\[5\] for hard ground strata, which is defined as Soil Type 1, are listed in Table 1.

### Table 1 Response spectrum parameters used in this study

<table>
<thead>
<tr>
<th>$a_0$ (m/s$^2$)</th>
<th>$S$</th>
<th>$\beta_0$</th>
<th>$T_B$ (s)</th>
<th>$T_C$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2</td>
<td>1.5</td>
<td>2.5</td>
<td>0.16</td>
<td>0.576</td>
</tr>
</tbody>
</table>

2.5 Description of wind turbine models

Previous study has shown that the ratio of mass of nacelle and blades to the total mass of the wind turbine system remains almost constant regardless of the rated power of wind turbines\[5\] as shown...
in Table 2. Therefore, in this study, two wind turbines with rated power of 400kW and 2MW are selected as typical examples for investigations. Details of basic parameters of these wind turbines are summarized in Table 3.

Table 2 Mass ratio of different wind turbines

<table>
<thead>
<tr>
<th>Model</th>
<th>Rated Power (kW)</th>
<th>Mass Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>0.44</td>
</tr>
<tr>
<td>2</td>
<td>400</td>
<td>0.46</td>
</tr>
<tr>
<td>3</td>
<td>500</td>
<td>0.45</td>
</tr>
<tr>
<td>4</td>
<td>1000</td>
<td>0.41</td>
</tr>
<tr>
<td>5</td>
<td>1500</td>
<td>0.45</td>
</tr>
<tr>
<td>6</td>
<td>2000</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Table 3 Details of the wind turbine models

<table>
<thead>
<tr>
<th>Description</th>
<th>Unit</th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated Power</td>
<td>kW</td>
<td>400</td>
<td>2000</td>
</tr>
<tr>
<td>Rotor diameter</td>
<td>m</td>
<td>31</td>
<td>80</td>
</tr>
<tr>
<td>Rotor tilt</td>
<td>deg</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Tower height</td>
<td>m</td>
<td>35</td>
<td>67</td>
</tr>
<tr>
<td>Hub height</td>
<td>m</td>
<td>36</td>
<td>67</td>
</tr>
<tr>
<td>Blade mass</td>
<td>kg</td>
<td>1100</td>
<td>6800</td>
</tr>
</tbody>
</table>

3. Proposal for estimation of seismic loads

Application of the response spectrum method to estimate seismic loads of the wind turbine support structure is hindered by two problems. Wind turbine support structures are significantly low damped, and response spectrum for such low damped structures show excessive fluctuation. Such uncertainty in response of these structures can not be captured by existing models of the damping correction factors defined in Eurocode\(^2\). Therefore, formulation of new damping correction factor is needed to incorporate extreme fluctuation in spectral acceleration to establish a reliable design spectrum for wind turbines. In addition, vertical distribution of the seismic loads is found to be largely affected by the higher modes\(^1\) of wind turbines. However, previous estimation of vertical load profile is based on the simplified SDOF model suggested by IEC\(^4\) that fails to capture the non-linearity of vertical load profiles. Therefore it is necessary to include higher modes in the load estimation procedure.

In this section, a damping correction factor that account for uncertainty in the response spectrum and modal participation function for higher modes are proposed for accurate estimation of the design loads.

3.1 Model for damping correction factor

To account for excessive fluctuations in the response spectrum of low damped systems, damping correction factor is proposed as a function of spectral uncertainty, natural period and damping ratio so that,

\[
F_r(\zeta, T, \gamma) = \left(\frac{7}{2+100\zeta}\right)^{\alpha}, \quad \alpha = f(T, \gamma)
\]

where \(T\) is natural period function, \(\zeta\) is damping ratio and \(\gamma\) is the quantile value for desired reliability level. Previously Eurocode defined damping correction factor as a function of damping ratio only and a constant value of 0.5 was used for the exponent \(\alpha\)\(^2\). However, when exponent \(\alpha\) is defined as a function of time period, \(f(T)\), it corresponds to a damping correction factor that considers the natural period of the structure\(^7\). In addition to damping ratio and natural period of the structure, proposed damping correction factor also includes uncertainty of the response spectrum.

To establish the proposed damping correction factor, first a set of seismic waves is generated to statistically evaluate the uncertainty in response spectrum of SDOF system. Then relation for exponent \(\alpha\) was established by data fitting to different quantiles of the response spectrum. Following describes the procedure in detail.
a. Probability distribution of uncertainty in Response Spectrum

A set of 35 seismic waves, 5 with observed phase and 30 with random phase, was used to evaluate excessive fluctuation in the acceleration response spectrum for range of damping ratios, i.e., from 0.5% to 5%. Figure 2 shows acceleration response spectra for damping ratios of 0.5% and 5% that correspond to wind turbine structures and buildings respectively. It can be observed that at low damping ratio of 0.5%, in case of wind turbine support structures, excessive fluctuations in the spectral acceleration occur.

![Fig. 2 Sections of acceleration response spectrums for statistical investigation](image)

![Fig. 3 CRF of spectral acceleration for each section](image)

To determine probability distribution that represents uncertainty involved, response spectrum is divided into three sections so that $0.05 < T < T_B$ refers to section $I_A$, $T_B \leq T \leq T_C$ refers to section $I_B$ and $T_C < T < 5$ refers to section $I_C$. Sections $I_A$ and $I_C$ that define the non-linear regions of the response spectrum are divided into ten sub-sections $I_{A1}^{(i)} \cdot I_{C1}^{(i)} (i = 1 \sim 10)$, whereas section $I_B$ was considered as a single section to calculate the statistical properties such as mean and standard deviation of acceleration response. Figure 3 shows a cumulative relative frequency in the sections $I_{A1}^{(i)} \cdot I_{B1}^{(i)} \cdot I_{C1}^{(i)}$. Also cumulative frequencies of lognormal distribution function derived from mean and standard deviation for each interval are drawn as solid lines. Log normal distribution is found to have well defined the uncertainty in all sections of the acceleration response spectrum as shown in above Fig. 3.

b. Identification of exponent $\alpha$ in damping correction factor

It is now possible to define the percent quantile $\gamma$ of acceleration for a desired reliability level by modeling the uncertainty of acceleration response with logarithm normal distribution function. Three quantile values of 20%, 50% and 80% are used to investigate exponent $\alpha$ of the damping correction factor. Percentile quantile value of acceleration spectrum is used for $S_a(T, \zeta)$ of equation (4) to calculate damping correction factor $F_\zeta$ and then exponent $\alpha$ is calculated using equation (5). The exponent $\alpha$ is found to have a linear relation with percent quantile $\gamma$ and natural period $T$, as shown in Fig. 4 and Fig. 5, that leads to following:

$$\alpha = f(T, \gamma) = -0.07T + 0.7\gamma + 0.5$$

(7)

The acceleration response spectrum calculated by proposed formula agrees well with the calculated results for all quantile values $\gamma$ of response acceleration well. Introduction of natural period $T$ of...
structure has lead to accurate estimation of response spectrum in the long period regions. Also uncertainty of the response spectrum can be incorporated by changing quantile value $\gamma$. However, for response spectrum based on the damping correction factor defined in Eurocode, it is found that it corresponds to 20% quantile values.

![Fig. 4 Variation of $\alpha$ with quantile $\gamma$](image1)

![Fig. 5 Variation of $T$ with quantile $\gamma$](image2)

![Fig. 6 Variation of response spectrum with $\gamma$-values](image3)

### 3.2 Vertical load distribution function considering higher modes

Seismic loads acting on wind turbine support structure can be calculated using load formula for MDOF system presented in equations (8) and (9). But estimation of design loads for MDOF system, shown in Fig 1, require prior knowledge of natural periods and modal participation function $\gamma_jX_{ij}$ for predominant modes. These structural characteristics are generally calculated by performing eigen value analysis. However, this study proposes use of natural period ratio and polynomial expression to estimate the natural period and modal participation function of higher modes. Then SRSS method is used for super positioning of these modes to obtain the maximum design load for wind turbines as shown in equation 10.

$$Q_n = \sum_{i=1}^{n} F_i = \sum_{i=1}^{n} F_i X_{ij} S_{ij} (T_j, \xi) m_i \quad (8)$$

$$M_n = \sum_{i=1}^{n} F_i (z_i - z_j) = \sum_{i=1}^{n} F_i X_{ij} S_{ij} (T_j, \xi) m_i (z_i - z_j) \quad (9)$$

$$Q = \sqrt{\sum_{j=1}^{n} Q_j^2} \quad M = \sqrt{\sum_{j=1}^{n} M_j^2} \quad (10)$$
a. Modeling of modal participation functions and time period ratio

Mode shape of wind turbines are generally calculated using eigen value analysis. Previous research [8] presented a polynomial expression as a function of height ratio for the modal participation function of first mode \( \gamma X_{ij} \) of wind turbine support structure. This study proposes polynomial equation as a function of height ratio to estimate the modal participation function for higher modes of the wind turbine support structures as shown below:

\[
\gamma X_{ij} = \sum_{k} c_{jk} \left( \frac{z_i}{H} \right)^k
\]

where \( z_i \) and \( H \) are the height of i-th node, hub height of wind turbine tower and \( c_{jk} \) are coefficients of polynomial respectively. The time period of higher modes are presented as a ratio of the natural period of respective mode to that of the first mode, e.g., \( T_j/T_1 \) is defined for period of the j-th mode.

Table 4 shows the coefficients \( c_{jk} \) and time period ratio \( T_j/T_1 \) for the first three modes.

<table>
<thead>
<tr>
<th>( j )</th>
<th>( k )</th>
<th>( c_{jk} )</th>
<th>( T_j/T_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>-6.00</td>
<td>0.127</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>-38.20</td>
<td>0.043</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.00</td>
<td>0.127</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>24.00</td>
<td>0.043</td>
</tr>
</tbody>
</table>

Table 3 Coefficients of polynomial and time period ratio

![Fig. 7 Comparison between the proposed and calculated \( \gamma X_{ij} \)](image)

Fig. 7 shows comparison between eigen and proposed modal participation function for first three modes of 400kW and 2MW wind turbine. From this figure, it is clear that modal participation function of different sized wind turbines can be calculated accurately by the proposed polynomial.

b. Estimation of seismic load for each mode and their combination

For load estimation by spectrum method, IEC61400 requires use of structural modes that account for the required total modal mass of 85%[4]. In this study, first three modes are found to satisfy this criterion. First shear load profile for selected modes are calculated using the proposed equations of modal participation function and time period ratio as shown in Fig 8. In case of 400kW turbine, contribution of higher modes is smaller compared to 1st mode. However, a significant contribution by 2nd and 3rd modes can be observed at the base of 2MW turbine.

![Fig. 8 Shear force profile of first three modes (\( \gamma = 0.5 \))]
Since consecutive modes of wind turbine are well separated, SRSS method is used for superposition of these modes as shown in equation(10). Figure 9 shows profile of shear force ratio of current study by SRSS method and time history analysis along with previous work[8] based on the 1st mode of vibration. It is clear that load profiles obtained from proposed equations could capture the non-linearity of vertical profile and show good agreement with time history analysis results.

Fig.8 Vertical profiles of shear force and bending moment ratios

3.3 Determination of quantile γ for current reliability levels

In section 3.1, quantile γ was introduced to the damping correction factor that accounted for large uncertainty of response spectrum. Therefore, it is necessary to determine suitable quantile value γ for defining reliable design spectrum. Based on reliability theory, code calibration method[9] is an effective approach to ensure essentially similar reliability level as that of current design codes. In Japan, BSL[4] requires time history analysis of structures for at least three earthquake waves to obtain structural design certification. These waves include two local waves, such as Kobe and Hachinohe that are onshore & offshore waves respectively, and one famous earthquakes like Elcentro or Taft. In this study, four of the above mentioned seismic waves are used to determine suitable quantile γ for determining the design response spectrum. The vertical load profiles obtained by time history analysis of selected wind turbines are shown in Fig. 9. A γ-value of 0.7, i.e., 70% quantile, is identified against the time history analysis results to obtain reliability level similar to that established by the current design code.

Fig. 9 Comparison of seismic load profiles: Time history analysis and proposed formula (γ = 0.5)
4. Conclusions

In this study, a modified damping correction factor is proposed that accounts for the excessive fluctuation of response spectrum at low damping ratios and consider natural period of the wind turbines. In addition, formula for analytical estimation of complex profile of seismic design loads are presented that introduce contribution of higher modes, up to third mode, to the vertical load distribution. Finally accuracy of proposed formula is verified against time history analysis and, reliability level similar to that established by the current design code is demonstrated using code calibration method.

References
5. JSCE. Guidelines for design of wind turbine support structures and foundations; Japanese society of civil engineers, 2007. (in Japanese)
8. Ishihara T, Zhu L, Binh LV. Earthquake load estimation method for the wind turbines under operation and parked conditions. 29th Wind Engineering Symposium: Tokyo, 2007; 187-190