A numerical study on the dynamic response of a floating offshore wind turbine system due to resonance and nonlinear wave

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ABSTRACT: A FEM code was developed to predict the dynamic response of a floating offshore wind turbine system in the time domain, employing the Morison’s equation and Srinivasan’s model to calculate the hydrodynamic drag forces and inertia forces on the floating structure, and quasi-steady theory to calculate the aerodynamic forces on wind turbines. The responses predicted by the proposed numerical model show good agreements with experiments. Morison’s equation provides satisfactory prediction for the surge motion, but overestimates the heave motion of the floating structure with vertical column. The drag force model proposed by Srinivasan et al. gives good predictions. Elastic deformation plays an important role in dynamic response of the floating structure. The predicted response was underestimated when the elastic deformation is ignored. The nonlinearity of wave becomes dominant for the water depth less than 100m. The elastic modes might be resonant with the higher order harmonic component of the nonlinear wave, resulting in the increase of the dynamic response of the floating structure.

KEYWORDS: Floating offshore wind turbine system, Morison’s equation, Srinivasan’s model, hydrodynamic damping, aerodynamic damping, elastic deformation, resonance, nonlinear wave.

1 INTRODUCTION

Accurate prediction of dynamic response of floating structures due to resonance and nonlinear wave is essential to evaluate the performance and to enlarge the applicability of the floating offshore wind turbine system when the ultimate limit plays a dominant role in the design at the intermediate water depth. There are two approaches to investigate the dynamic response of the floating structures. One is the application of the Morison’s equation [1], and the other is the linear potential theory [2]. The linear potential theory is raised from the assumptions of zero viscosity of the fluid, while the Morison’s equation considers the effect of the viscosity represented by the nonlinear hydrodynamic drag force. After FLOAT project [3], several floating offshore wind turbine systems (see, [2], [4], [5], [6]) have been proposed based on the concepts in the oil and gas industry such as semi-submersible, spar and tension leg platform. Some preliminary studies have been done to assess the platform using the numerical models that applied in the design of the platform used by oil and gas industry in the deep water. Hederson et al. [6] used the Morison’s equation for the floating structures with large diameter sub-structures. They ignored the hydrodynamic drag force and solve the linearized equation of motion in the frequency domain. This simplification might lead to some loss in accuracy to predict the dynamic response of the floating structures with small diameter sub-structures, such as the floater proposed by Ishihara et al. [7] for the economical design.

In the present study, a FEM code with Morison’s equation and Srinivasan’s model was developed in the time domain to investigate the effects of the hydrodynamic damping and the nonlinear wave on the response of the floating structures. The performance of the code for the dynamic response analysis of the semi-submersible floating structures was verified by a model test.
2 NUMERICAL METHOD OF FULL DYNAMIC SIMULATION

2.1 Governing equation

The general formulation of the differential equation of motion for a floating offshore wind turbine system can be written as

\[
[M]{\ddot{X}} + [C]{\dot{X}} + [K]{X} = \{F_G\} + \{F_R\} + \{F_H\} + \{F_W\}
\]  

(1)

where \([M]\) is a mass matrix, \([C]\) is a damping matrix, \([K]\) is a stiffness matrix of structure, \(X\) and its derivatives are unknown vectors of 6 degree of freedom (3 translations and 3 rotations) and their derivatives. The terms in right side of the equation (1) are the external force vectors acting on the system and typically varies with time, where \(\{F_G\}\) is the mooring force, \(\{F_R\}\) is the hydrostatic restoring force, \(\{F_H\}\) is the wave exciting force, \(\{F_W\}\) is the aerodynamic force.

The mooring force is defined as follows,

\(\{F_G\} = -[K_G]{X}\)

(2)

where the mooring stiffness \([K_G]\) is determined by a result of catenary’s analysis from the steady forces, including the tidal current force, wind force and wave drift force.

Under assumption of infinitesimal displacement theory, hydrostatic restoring force can be simplified by the first-order hydrostatic restoring force coefficient \([R]\) as follows.

\(\{F_R\} = -[R]{X}\)

(3)

where \(\rho_w\) is the density of water, \(g\) is the gravity acceleration, \(A_w\) is the still surface area, \(W\) is the weight of the model, \(G_{M_x}\) and \(G_{M_y}\) are the meta-center height in \(X\) and \(Y\) direction, respectively.

In order to calculate the wave exciting force on the floating structure, the modified Morison’s equation by Sarpkaya et al. [9] can be adapted as shown in the equation (5).

\[
F_H = F_D + F_{EW} + F_{EM} = 0.5\rho_wC_D A |u - \dot{X}|(u - \dot{X}) + \rho_w C_M V\dot{u} - M_a \ddot{X}
\]

\[M_a = \rho_w (C_M - 1) V
\]

(5)

where \(F_D\) is drag force, \(\rho_w\) is water density; \(u\) is particle wave velocity, \(A\) and \(V\) are the area and volume of the element, \(M_a\) is called as added mass coefficients, \(\dot{X}\) is the velocity of the moving element, \(C_D\) and \(C_M\) are hydrodynamic drag and inertia coefficients, respectively.

The drag and inertia coefficients depend on the cross-sectional shape of the structure, which was commonly given as the functions of the Keylegan-Carpenter number \(K_c = u_{max}T/D\) as shown by Offshore Standard DNV-OS-J101[10], where \(u_{max}\) is the maximum wave particle velocity at still water level, \(v\) is the kinematic viscosity of water, \(T\) is the period of the waves. Since the relative wave particle velocity as well as the first term of the
wave exciting force contains the velocity of the moving element, the hydrodynamic damping is automatically taken into account during the simulation.

Since the Morison’s equation cannot predict forces acting on the bottom of the vertical cylinder accurately, the added inertia force acting on the bottom of the base floater proposed by Haslum [11] and the drag force proposed by Srinivasan [12] are used, as shown in equations of (7) and (8),

\[ M_a = \rho_a \left( C_M - 1 \right) 2 \pi 3 (D/2)^3, \quad C_M = 2.0 \] (7)

\[ F_D = -C_{ED} \dot{X}_3; \quad C_{ED} = 2 \zeta \omega (M_3 + M_{a3}); \quad (\zeta = 20\%) \] (8)

where \( D \) is the diameter of the bottom of the base floater, \( \dot{X}_3 \) is the velocity of the moving element in the vertical direction, \( \omega \) is the angular frequency of heave mode, \( M_3 \) and \( M_{a3} \) are the structure mass and hydrodynamic added mass of floater in the heave direction, respectively.

The quasi-steady aerodynamic theory is used in the calculation of the aerodynamic forces, in which the drag, the lift force and the moment are estimated by using aerodynamic coefficients and the relative wind speed as follows.

\[ \{F_W\} = \{F_D, F_L, F_M\} = \left\{0.5 \rho d C_D (\alpha) V^2, 0.5 \rho d C_L (\alpha) V^2, 0.5 \rho d^2 C_M (\alpha) V^2\right\} \] (9)

where \( \rho \) is the density of the air, \( d \) is a reference dimension of the wind turbine, \( C_D \) is the aerodynamic drag coefficient, \( C_L \) is aerodynamic lift coefficient, \( C_M \) is aerodynamic moment coefficient, \( \alpha \) is angle of attack of the relative wind speed \( V \). Here, the relative wind speed with respect to the moving element can be written as

\[ V = U - \dot{X} \] (10)

where \( U \) is the wind velocity, \( \dot{X} \) is the velocity of the moving element. Since the relative wind speed as well as aerodynamic force terms contains the velocity of the moving element, the aerodynamic damping is automatically taken into account during the simulation.

2.2 Numerical scheme

In this study, the mooring force, hydrostatic restoring force and the added inertia force were moved to left side of the equation to solve as follows,

\[ \left\{ [M] + [M_a] \right\} \dot{X} + \left\{ [C] + [C_D] \right\} \dot{X} + \left\{ [K] + [K_G] + [K_R] \right\} X = \{F_D\} + \{F_{EW}\} + \{F_W\} \] (11)

A FEM code based on above equation was developed to predict the eigenperiods and dynamic responses of the floating offshore wind turbine system. A brief description of the code is summarized in Table 1. The beam elements were used for the discretization and the mass of each element was concentrated at its nodes constructing a symmetrical lumped mass matrix.

The damping matrices is defined by Rayleigh damping method [13] and it can be written as follows,

\[ [C] = \alpha [M] + \beta [K] \] (12)

\[ [C_D] = \alpha_l \left( [M] + [M_a] \right) + \beta_l [K] \] (13)

where \( \alpha, \beta \) is the function of the eigen-periods and structural damping ratios, \( \alpha_l, \beta_l \) is the function of the eigen-periods and damping ratios of the system in the vertical direction.
Table 1. A brief description of the FEM code

<table>
<thead>
<tr>
<th>Dynamic analysis</th>
<th>Direct numerical integration, the Newmark method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eigenvalue analysis</td>
<td>Subspace iteration procedure</td>
</tr>
<tr>
<td>Element type</td>
<td>Beam element</td>
</tr>
<tr>
<td>Formulation</td>
<td>Total Lagrangian formulation</td>
</tr>
<tr>
<td>Damping</td>
<td>Rayleigh damping</td>
</tr>
</tbody>
</table>

3 NUMERICAL RESULTS

A semi-submersible floating structure as shown in Figure 1 was modeled with beam elements including 186 nodes and 188 elements in order to evaluate the performance of the developed FEM code. Here, the mooring was simplified by the longitudinal linear spring. The eigenvalue analysis was carried to calculate the mode of the floating structure and the eigenperiods were compared with the natural periods obtained from the decay test [7]. The dynamic response analysis was also conducted in the same condition of the experiment and the predicted responses were compared with those from the water tank test.

![Figure 1 A semi-submersible FOWTS modeled with beam elements](image)

In the analysis, the hydrodynamic drag and inertia coefficients were defined as the functions of Kevlegan-Carpenter number recommended by Offshore Standard DNV-OS-J101 [10]. Here, these coefficients of the rectangle connecting girders were modified by the ratio between the rectangle and the cylinder column described by Motora et al. [8]. The Kevlegan-Carpenter number was simplified by the incident wave height and diameter of column by the equation (14).

\[
K_c = \frac{UT}{D} = \pi \frac{H}{D}
\] (14)

For the calculation of the aerodynamic force, the aerodynamic drag coefficient of 0.6, 0.6 and 1.3 was used for the tower, nacelle and blades, respectively in the survival condition, and the rotor was modeled by the thrust acting on the hub of the wind turbine with thrust coefficient \( C_T = 0.33 \) corresponding to the operating condition. The wave particle velocities and accelerations were generated by the Airy theory for the linear wave and the stream function for the nonlinear wave.

A test with 1:150 scale model was carried out under Froude similarity law in the water tank with wind tunnel owned by the National Maritime Research Institute of Japan [7]. The motion of the central column and the strains...
of the horizontal brace were measured. The strains were used to calculate the excited bending moment on the brace. Experimental conditions are shown in Table 2.

Table 2 Experimental conditions

<table>
<thead>
<tr>
<th>Wind turbine condition</th>
<th>Prototype</th>
<th>1:150 scale model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U (m/s)</td>
<td>H (m)</td>
</tr>
<tr>
<td>Operating</td>
<td>3-25</td>
<td>1.5-4.6</td>
</tr>
<tr>
<td>Extreme</td>
<td>50</td>
<td>12.0</td>
</tr>
<tr>
<td>Experimental condition</td>
<td>0.25,50</td>
<td>3,6,12</td>
</tr>
</tbody>
</table>

Environmental condition (U: wind speed, H: wave height; T: wave period)

3.1 Eigenvalue analysis

A semi-submersible FOWTS was modeled with beam elements including 186 nodes and 188 elements. The eigenvalue analysis was carried out to calculate the mode of the FOWTS and the results were compared with the natural periods obtained from the decay test [7]. The eigen-periods in Table 3 show good agreement with the experiment in the surge, heave and pitch direction. The higher mode corresponding to the elastic deformation was also obtained.

Table 3. Comparison of measured and predicted natural periods

<table>
<thead>
<tr>
<th>Mode</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural period (s)</td>
<td>36.7</td>
<td>34.0</td>
<td>33.3</td>
<td>-</td>
</tr>
<tr>
<td>Eigen-period (s)</td>
<td>36.0</td>
<td>34.0</td>
<td>33.0</td>
<td>7.3</td>
</tr>
</tbody>
</table>

3.2 Hydrodynamic damping effect

Figure 2 presents the variations of normalized responses with the wave periods obtained from the numerical simulation and the experiment. As expected, the responses of the surge motion vary linearly with the wave height in the ranges of periods far from the eigen-periods, but they show significant peak around the eigen-period and decrease significantly when wave heights increase. The predicted responses show good agreement with the experiment in the nonresonant region. The Morison’s equation provides a good prediction for the surge motion, but it overestimates the responses when the drag force is ignored in the resonant region. The Morison’s equation is overestimated when the only Morison’s equation was applied, while the predicted heave motions by the Srinivasan’s model show good agreement with experiments.

Figure 3 shows variations of responses and damping ratios with the wave height. The equivalent hydrodynamic damping ratios increase with the increase of the wave height. This is why the surge and heave motions decrease for higher wave decrease.
3.3 The aerodynamic damping effect

A dynamic response analysis was also carried out to investigate the effect of wind turbine on the floating structure in the operating and the survival conditions. Figure 4(a) presents variations of responses of the surge motion with the wave periods in the operating condition. As the experiment [7], the predicted peak responses around the resonant period in the windy day were less than that in the calm day, and the effect of the aerodynamic damping is more significant for the cases with smaller wave height, having lower hydrodynamic damping. Figure 4(b) shows the variations of responses of the surge motion in the survival condition. There is little dependency on the wind speed. This is because the aerodynamic damping from the blade is much smaller than the hydrodynamic damping.

3.4 The effect of elastic deformation

To investigate the effect of elastic deformations to the dynamic response of the floating structure, two models with the beam elements were constructed using the structural properties of the prototype floating structure including the number of elements and nodes as the same as those used in the previous section. The SM570 steel material defined by Architecture Institute of Japan [14] was used for the horizontal brace and others were rigid in elastic model. The structural damping ratio of the horizontal brace was 0.8% for the steel material as mentioned by Burton et al. [15]. The dynamic response analyses for both models were performed for the regular wave periods from 1.0s~30.0s with the wave height of 12m in the survival condition. The incident wave direction was -90 degree.
Figure 5 shows the variations of the excited bending moments with the wave periods for the rigid and elastic models. The bending moments for both models show good agreements with the experiments in the nonresonant region, but the rigid model underestimates the responses in the resonant region. The peaks of response found at the period near 7 seconds correspond to the eigenperiods of the elastic model.

![Figure 5: Variations of excited bending moments with the wave periods](image)

**Figure 5** Variations of excited bending moments with the wave periods

3.5 The nonlinear wave effect

Waves at reference sites of water depth 50,100,200m with the maximum wave height $H=22.23m$ and wave period $T=15.5s$ were selected to investigate the characteristics of the nonlinear wave. Figure 6 and Figure 7 show regular wave theory selections and water elevations for several sea depths, respectively. The wave at the water depth of 50m near water breaking limit requires to use the 9th order of the stream function. At 100-200m sea depth, 1st harmonic component of the wave elevation is dominant, while about 25% of the wave elevation is contributed by 2nd harmonic component at 50m depth.

![Figure 6: Regular wave theory selections](image)

![Figure 7: Water elevations for several sea depths.](image)

A dynamic response analysis of the floating structure was carried out for these reference sites. The applied load and surge-exciting bending moment on the horizontal brace was investigated. The applied load in surge direction at 50m depth is larger than the other cases and has the second harmonic component as shown in Figure 8.

As a result, the surge-exciting bending moment at 50m depth is 1.8 times larger than the other cases, as shown in Figure 9. The Fourier harmonic components of the surge-exciting bending moment indicate that the second harmonic component for 50m sea depth has a large contribution to the response of the floating structure. This is because the second harmonic component of wave ($T=7.8s$) resonates with the elastic mode ($T=7.3s$) of the floating structure.
4 SUMMARY AND CONCLUSIONS

A FEM code with the Morison’s equation and Srinivasan’s model was developed in the time domain to investigate the effects of the hydrodynamic damping and the nonlinear wave on the response of the floating structures. A dynamic response analysis in the intermediate depth was also carried to clarify the effect of the nonlinear wave on the response of the floating structure. The following results were obtained.

1) The predicted eigenperiods show good agreement with experiment in the surge, heave and pitch direction.
2) The response is strongly influenced by hydrodynamic damping in the resonance region, but it is not affected much in other regions. Morison’s equation provides satisfactory prediction of the surge motion, but overestimates the heave motion of the floating structure with vertical column, while the drag force model proposed by Srinivasan gives good prediction for the heave motion.
3) The dependency of the peak surge on the wind speed in the operating condition was observed due to the aero-dynamic damping for the wind turbine, but it is not in the survival condition. It indicates that the surges around the resonant period will be overestimated in the operating condition when the interaction between wind turbines and floating structures are neglected.
4) Elastic deformation plays an important role in dynamic response of the floating structure. The predicted response is underestimated when the elastic deformation is ignored.
5) The nonlinearity of wave becomes dominant for the water depth less than 100m, and the elastic modes might be resonant with the higher order harmonic component of nonlinear wave, resulting in the increase of dynamic response of the floating structure.

REFERENCES