A study of dynamic wake model for prediction of wind farm power production
Part 1: A novel double Gaussian wake model considering yaw misalignments

ウインドファーム発電量予測のためのダイナミックウェイクモデルに関する研究
その１　ヨーミスアライメントを考慮した新しいダブルガウシアンウェイクモデルの提案

Takeshi ISHIHARA [1)]　　Yunpeng SONG [2)]

○石原　孟[1)]　　宋　雲鵬[2)]

1. Introduction

Wake flow causes problems such as reduced power generation and increased fatigue loads in downstream wind turbines. It is important to develop analytical models that allow fast and accurate prediction of power production in large wind farms. In previous research, wake deflection was evaluated using single Gaussian wake models which only focused on far wake prediction and ignored the double-peak wake deficit in the near wake region. This results in the inability of single Gaussian models to accurately predict wake deficit in the near wake region[1)], and makes traditional wake models difficult to accurately evaluate the wake of wind turbines in wind farms with dense layouts and narrow wind turbine spacing, such as some onshore wind farms in Japan, where 2D spacing are frequently observed.

In this study, an analytical wake model to predict both near and far wake deflection is derived from a double-Gaussian wake model which improve near wake accuracy. The proposed analytical model is then validated by numerical simulations.

2. A new analytical model for yawed wind turbine

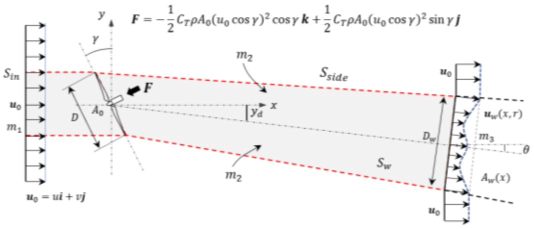

Figure 1. Schematic of the momentum conservation-based model for the wake deflection

The fully developed wake flow behind wind turbines has been investigated for decades. In the previous study, the assumption of axisymmetric and the self-similar distribution for the velocity deficit in the far wake region were inherited. In this study, the double Gaussian shaped velocity deficit is adopted together with the momentum conservation streamwise and spanwise to derive an analytical deflection model.

Figure 1 shows the schematic of of the momentum conservation-based model for the wake deflection. In control volume, since they're only rotor thrust force $\boldsymbol{F}$ acts as the external force, the governing equations can be established by the steady-state form of the NS equation as:

$$\rho\int_{S_w}\boldsymbol{u}_w\cdot\boldsymbol{n}dS - \left(\rho\int_{S_{side}}\boldsymbol{u}_0\cdot\boldsymbol{n}dS + \rho\int_{S_{in}}\boldsymbol{u}_0\cdot\boldsymbol{n}dS\right) = 0 \qquad (1)$$

$$\rho\int_{S_w}\boldsymbol{u}_w\boldsymbol{u}_w\cdot\boldsymbol{n}dS - \left(\rho\int_{S_{side}}\boldsymbol{u}_0\boldsymbol{u}_0\cdot\boldsymbol{n}dS + \rho\int_{S_{in}}\boldsymbol{u}_0\boldsymbol{u}_0\cdot\boldsymbol{n}dS\right) = \boldsymbol{F} \qquad (2)$$

where u_w donate wake velocity, u_0 represent freestream wind speed. The turbine-induced force F exerting on the control volume can be expressed by the following equation:

$$\boldsymbol{F} = -\frac{1}{2}C_T\rho A_0(u_0\cos\gamma)^2\cos\gamma\,\boldsymbol{k} + \frac{1}{2}C_T\rho A_0(u_0\cos\gamma)^2\sin\gamma\,\boldsymbol{j} \qquad (3)$$

where C_T represents the thrust coefficient of the wind turbine, γ is the yaw misalignment, A_0 donate the rotor area of the wind turbine. Equation (3) can be then divided into Equations (4) and (5) in the streamwise and spanwise directions as:

$$\rho\int_{S_w}u_w{}^2 - u_0u_w dS \approx -\frac{1}{2}\rho A_0C_T(u_0\cos\gamma)^2\cos\gamma \qquad (4)$$

$$\sin\theta\,\rho\int_{S_w}u_w^2\mathrm{d}S = \frac{1}{2}\rho A_0C_T(u_0\cos\gamma)^2\sin\gamma \qquad (5)$$

Following the self-similarity assumption, the velocity in the wake region can also be decomposed into streamwise and spanwise components respectively, where a double Gaussian shape function for the spanwise profile is used:

$$u_w(x,r) = u_0(1 - F(x)\varphi(r)) \qquad (6)$$

$$\varphi(r) = \frac{1}{2}[\exp D_+ + \exp D_-], \qquad D_\pm = -\frac{(r \pm r_{min})^2}{2\sigma^2} \qquad (7)$$

where r is the radius away from the trajectory line spanwise, r_{min} represents the distance of the center offset of double Gaussian distribution.[2)] Substitute equation (6) back to Equation (4) and adopt a double Gaussian shape function, the following relation can be derived by considering second order Taylor expansion:

$$F = \frac{M - \sqrt{M^2 - 1/2N\cos^3\gamma D^2C_T}}{2N} \cong \frac{C_T'D^2}{8M} + \frac{NC_T'^2D^4}{64M^3} \qquad (8)$$

where $C_T' = C_T\cos^3\gamma$, M and N can be expressed as:

1）　東京大学大学院工学系研究科　教授
Professor, The University of Tokyo

2）　東京大学大学院工学系研究科　大学院生
Graduate Student, The University of Tokyo

$$M = 2\sigma^2 \, exp(-\frac{r_{min}^2}{2\sigma^2}) + \sqrt{2\pi} r_{min} \sigma \, erf\left(\frac{r_{min}}{\sqrt{2}\sigma}\right) \tag{9}$$

$$N = \sigma^2 \, exp(-\frac{r_{min}^2}{\sigma^2}) + \frac{1}{2}\sqrt{\pi} r_{min} \sigma \, erf\left(\frac{r_{min}}{\sigma}\right) \tag{10}$$

In addition, as the double Gaussian distributions in the neighborhood converge into one to the far side, the radial peak position r_{min} is modeled using the following equation:

$$\frac{r_{min}}{R} = \frac{r_0}{2R}\left(1 - \mathrm{erf}\left(\frac{x - x_0}{\sqrt{2}mD}\right)\right) \tag{11}$$

where R is the rotor radius. In streamwise, linear wake expansion is assumed, thus σ can be expressed as:

$$\sigma = k^* x + \epsilon D \tag{12}$$

where $\epsilon = (1/2 - r_0/D)/3$, k^* is modeled using a function of C_T and I_a. Parameters of proposed formulas are then identified based on the results of numerical simulations using the Genetic Algorithm by Ishihara & Qian[2] as:

$$r_0 = 0.52R \tag{13}$$

$$m = 4.0 \tag{14}$$

$$x_0 = 3C_T^{0.65} I_a^{-0.31} D \tag{15}$$

$$k^* = 0.11 C_T^{0.65} I_a^{0.27} \tag{16}$$

Since the skew of wake trajectory is small, the approximation of $\sin\theta \approx \theta$ is adopted for simplification. Then spanwise function (5) can be rewritten as:

$$\theta = \frac{\frac{1}{8}\pi\rho D^2 C_T (u_0 \cos\gamma)^2 \sin\gamma}{\rho u_0^2 \int_{S_w} (1 - F\varphi)^2 dS} \tag{17}$$

Assume the wake radius as $R_w = D_w/2 = S_d\sigma$, then the integration part of the denominator of equation (17) can be expressed as:

$$\int_{S_w} (1 - F\varphi)^2 dS = 2\pi\left\{\frac{1}{2}R_w^2 - F\int_0^{R_w}(2\varphi - F\varphi^2) r dr\right\} \tag{18}$$

In previous study, Qian & Ishihara[3] suggested using scaling factor $S_d = 2\sqrt{2\ln 2}$. In this study, the scaling factor $S_d = 3.75\sqrt{2\ln 2}$ is used to obtain reasonable deflection that best fits the experimental data.

By substituting equation (7) and equation (8) into (18), the expression of skew angle can be finally derived as:

$$\theta(x) = \frac{C_T D^2 \cos^2\gamma \sin\gamma}{8R_w^2 - 32HF + 16KF^2} \tag{19}$$

$$H = \frac{1}{a}\exp(-a r_{min}^2) + r_{min}\sqrt{\frac{\pi}{a}}\left[\mathrm{erf}\left(\sqrt{a}(R_w - r_{min})\right) - \mathrm{erf}(\sqrt{a} r_{min})\right] - \frac{1}{2a}\left[\exp(-a(R_w - r_{min})^2) + \exp(-a(R_w + r_{min})^2)\right] \tag{20}$$

$$K = \frac{1}{2a}\exp(-2a r_{min}^2) + \frac{r_{min}}{2}\sqrt{\frac{\pi}{a}}\left[\mathrm{erf}\left(\sqrt{2a}(R_w - r_{min})\right) - \mathrm{erf}(\sqrt{2a} r_{min})\right] - \frac{1}{8a}\left[\exp(-2a(R_w - r_{min})^2) + \exp(-2a(R_w + r_{min})^2)\right] - \frac{1}{4a}\exp(-2a r_{min}^2)\exp(-2aR_w^2) \tag{21}$$

where $a = 0.5\sigma^2$ for simplification. Since it is not able to find an analytical solution for equation (19), wake deflection $y_d(x)$ is integrated numerically as:

$$\frac{y_d(x)}{D} = \frac{1}{D}\int_0^x \theta(x)\, dz \tag{22}$$

3. Model validation

A numerical simulation[3] with $C_T = 0.84$ and $I_a = 3.5\%$ in $8°$ and $16°$ yaw misalignments are conducted to validate the proposed analytical model. As shown in Figure 2, the proposed model not only accurately predicts the double-peak shaped horizontal wake profile in the near weak region, but also shows better performance in the far wake region. In terms of yaw deflection, the proposed model shows better agreement in the near wake region, while the single Gaussian weak model underestimates the deflection in the near wake region.

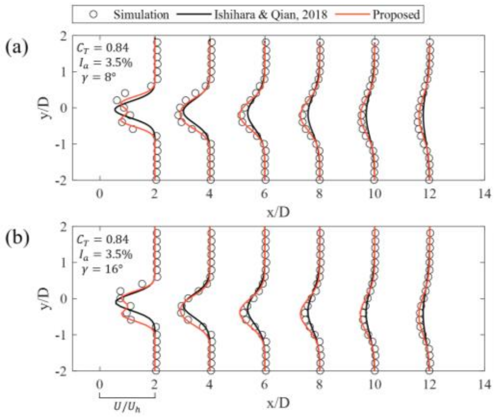

Figure 2. Comparison of the predicted distribution of mean wind speed in the wake (a) $\gamma = 8°$, (b) $\gamma = 16°$

In terms of wind power prediction for downstream wind turbines based on rotor-averaged wind speed $\bar{U}_{rot}$, the proposed model also shows better performance in both near and far wake regions in comparison with exists model as shown in Figure 3.

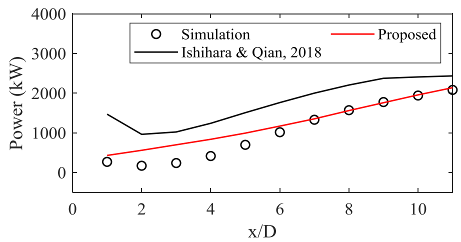

Figure 3. Downstream wind power production in the wake of a wind turbine

4. Conclusion

In this study the following conclusions are obtained:

(1) A new analytical wake deflection formula is proposed based on a double-Gaussian wake model.

(2) Proposed analytical model is validated by numerical simulation and shows favorable accuracy in the near and far wake regions.

References

1) Qian, G.-W. and Ishihara, T. A New Analytical Wake Model for Yawed Wind Turbines. *Energies*, 2018.

2) Ishihara, T. and Qian , G.-W. , A novel double Gaussian wake model considering the load distribution on rotor and turbulence mixing effects, *44th Wind Energy Symposium*, 2022. (In Japanese)

3) Ishihara, T., and Qian, G. W. A new Gaussian-based analytical wake model for wind turbines considering ambient turbulence intensities and thrust coefficient effects. *JWEIA*, 177, 275-292, 2018.