A study of dynamic wake model for prediction of wind farm power production
Part 2: A prediction model for wake advection velocity

ウインドファーム発電量予測のためのダイナミックウェイクモデルに関する研究
その２ ウェイク移流速度の予測モデルの提案

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1. Introduction

The steady-state wake model has been widely used for wind farm power prediction and control. However, the steady model does not consider dynamic wake behavior, such as wake advection processes, which is critical for real-time wind farm power prediction and model-predictive wind farm control. To overcome this limitation, there is a demand for an analytical wake model that can capture the dynamic effects of wake propagation without increasing significant costs. To achieve this, it is essential to accurately evaluate wake advection velocity U_c for dynamic wake advection processes. Several models have been proposed in the previous research to predict the wake advection processes in wind energy field[1), 2)]. Gebraad and Wingerden (2014) proposed a state-space model using the free-stream wind speed $\bar{U}_0$ to consider wake advection. [1)] Foloppe et al. (2022) later modified this approach by utilizing the spatial average wake velocity $\bar{U}_w$ to model wake advection. However, both models overestimated wake advection velocity, leading to inaccurate power production predictions.[2)]

In this study, a double Gaussian wake model is first adopted together with a space-state scheme for unsteady wake prediction. Then an empirical wake advection velocity model is proposed based on numerical simulation. Finally, the proposed model will be validated in 2 wind turbine layouts in terms of power production.

2. Methodology

1.1 Analytical model for dynamic wake prediction

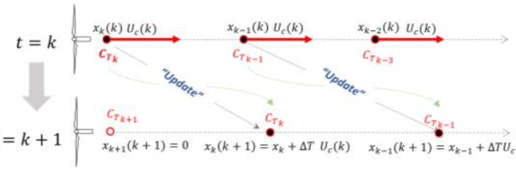

Figure 1 Space-state framework of the analytical model

To model the unsteady wake model, the space-state model with the observation point approach proposed by Gebraad and Wingerden (2014)[1)] is utilized to model the wake advection process. As shown in Figure 1, the position of each wake frame is updated as:

$$x_k(n) = x_k(n-1) + U_c(n-1)\Delta T \tag{1}$$

$$y_k(n) = y_k(n-1) + \Delta y_{c,k}(n) \tag{2}$$

where n is the time step, k represents the frame index, x and y donate the position of the existing wake frame streamwise and spanwise, U_c is the wake advection velocities in the main wind direction that transport the wake frame downstream. In Gebraad and Wingerden (2014), Taylor frozen hypothesis was used which consider $U_c = U_0$, while Foloppe et al. (2022) modified advection velocity as averages wind speed in wake region spanwise $U_c = \bar{U}_W = 1/A_W \int_{Wake} U(y,z)dA_W$, where A_W is the wake area. Since both the Taylor frozen hypothesis and averaged wake approaches overestimated wake advection velocity, an empirical model will be proposed in Section 3.1 and validated in section 3.2.

For each frame, the wind turbine thrust coefficient $C_{T,k}$ and yaw misalignment γ_k is conserved. For frame at $x_k = 0$, $C_{T,k}$ is calculated by using the look-up table of wind turbine rotor average wind speed $\bar{U}_{Rot,k}$ and yaw misalignment γ_k as:

$$C_{T,k} = f_{LUT}(\bar{U}_{Rot}, \gamma_k), \qquad \bar{U}_{Rot} = \frac{1}{A}\int_{Rotor} U(y,z,n)dA \tag{3}$$

Then, the velocity deficit $\Delta U(n)$ and wake deflection $\Delta y_{c,k}(n)$ in the wake frame k can be calculated by the double Gaussian wake model proposed by Ishihara and Qian[3)] considering the yaw misalignment.

2.2 Numerical Model

The finite volume method is applied for the discretization of the governing partial differential equations. The averaged continuity and momentum equations for incompressible flow with external aerodynamic forces of the wind turbine to simulate unsteady wake can be expressed as：

$$\frac{\partial(\rho\bar{u}_i)}{\partial x_i} = 0 \tag{4}$$

$$\frac{\partial(\rho\bar{u}_i)}{\partial t} + \frac{\partial(\rho\bar{u}_j\bar{u}_i)}{\partial x_j} = -\frac{\partial\bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial\bar{u}_i}{\partial x_j} + \frac{\partial\bar{u}_j}{\partial x_i}\right)\right] + \frac{\partial\tau_{ij}}{\partial x_j} + f_{rot,i} \tag{5}$$

where, u_i donate the velocity components in i direction, p donate pressure, ρ is the air density, μ is the molecular viscosity, ADM-R model has been used in this study for accounting rotor aerodynamic force $f_{rot,i}$. In this study, RSM embedded in ANSYS Fluent is selected to calculate

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the Reynolds stress tensor to close the momentum equation. To investigate the behavior of wake advection velocity, a triangular yaw maneuver is performed on simulation as shown in Figure 2, with a yaw range of 0° to 15°, and a yaw rate of 0.3°/s following commercial applications for safety considerations. The purpose of the yaw maneuver is to generate a meandering wake, which will be subjected to further analysis.

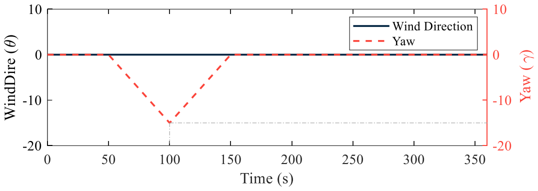

Figure 2 Yaw maneuver to generate a meandering wake.

The parameters used in the numerical simulations for each case are summarized in Table 1.

Table 1. Case setting for numerical simulation.

Case	Model	I_a	C_T
1	RSM	3.5%	0.84
2		3.5%	0.36
3		13.7%	0.84
4		13.7%	0.36

3. Results and Discussion

1.2 Wake advection velocity

The advection velocity can be defined as the velocity of downstream propagation of wake meandering[4]:

$$U_c = \frac{\Delta x}{\Delta T_c} \tag{6}$$

where Δx donates the distance between two sections, ΔT_c is the time delay of deflected wake pass through two measuring points. Following this approach, the advection velocity $U_c(x)$ in the wake region of the wind turbine in all 4 cases are identified as shown in Figure 3. Comparing with results identified from numerical simulation, both existing modelsoverestimate wake advection velocity.

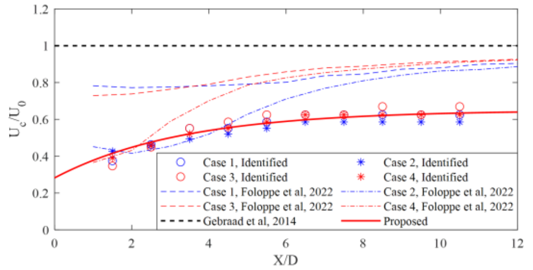

Figure 3 Advection factor alone downstream in comparison with numerical simulation results.

By assuming a homogenous advection process in cross-section, the advection velocity of the wake pattern can be assumed as the function of $\hat{x} = x/D$:

$$C(\hat{x}) = \frac{U_c(\hat{x})}{U_0} = -e^{a\hat{x}+b} + c \tag{7}$$

where $C(x)$ is the ratio of advection velocity to ambient velocity U_0. In this study, the parameters $a = -0.3$, $b = -1$, $c = 0.65$ are identified by using Genetic Algorithm. Finally, equation (7) is adopted to equation (1) for dynamic wake prediction.

3.2 Model Validation

To validate the proposed analytical model for unsteady power production prediction, a benchmark simulation of 2 wind turbines with a 7D spacing layout are conducted. As shown in Figure 4, the proposed model well predicts the delayed power increases induced by yaw steering of the upstream wind turbine, while the conventional models underestimate time delay.

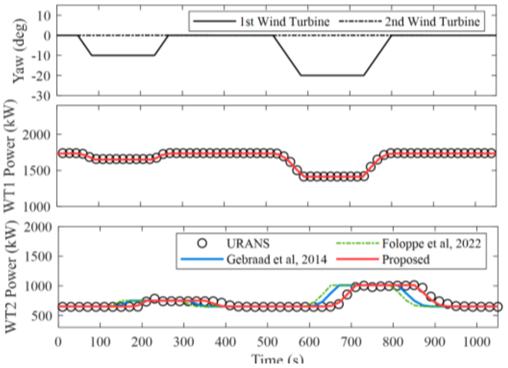

Figure 4 Power production comparison under yaw steering

In addition, the proposed model also shows the lowest relative RMSE to URANS benchmark simulation compared with the existing approaches as shown in Figure 5.

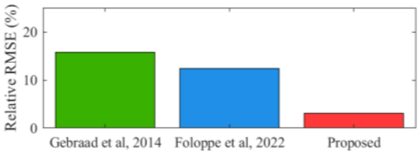

Figure 5 Relative RMSE to numerical simulation

4. Conclusion

In this study the following conclusions are obtained:

1) An analytical model for dynamic wake behavior is developed by combining the double Gaussian wake model with the space-state approach and a proposed wake advection velocity model.
2) The proposed framework is then validated by numerical simulation and shows better performance in the prediction of wind turbine power production compared with the existing approaches.

References

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