A new multiple wake model and its application to yaw – based wind farm control

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1. Introduction

In the wind farm, wakes from multiple turbines lead to a significant wake-turbine interaction as well as the wake-wake interaction, which reduces the whole energy output of the farm. The commonly used approach in wind farm design is to combine the single wake calculation based on the superposition approach for mean flow field, however, the turbulence was not considered in the multiple wake modelling [1,2]. In the IEC61400-1 for wind turbine design [3], the added turbulence from neighboring turbines are combined for fatigue prediction, while turbulence distribution is assumed constant in the wake and the accuracy of turbulence superposition has not been evaluated. Note that the variation of turbulence has a significant impact on the wake recovery and interactions. Recently, the concept of yaw-based wind farm control is proposed by coordinating the yaw control operations across the wind turbines to mitigate the wake losses [1, 4], however the effects of yaw offset limit were not investigated. On the other hand, the maximum yaw misalignment of 15° is required by IEC standard for safety consideration [3], which should be considered in the yaw control. This paper aims at proposing a new multiple wake model and then apply it to yaw-based wind farm control for maximizing the annual energy production (AEP).

2. A new multiple wakes model

2.1 Rotor effective onset

Downstream turbines in the wind farm experience a non-uniform inflow over the rotor area, hence in order to apply the wake models, an effective onset wind speed $U_{h,i}$ and local turbulence intensity $I_{a,i}$ on the rotor is evaluated as follows:

$$U_{h,i} = \frac{1}{A}\int_{rotor} U(x_i, y, z)\,\mathrm{d}A \tag{1}$$

$$I_{a,i} = \frac{1}{AU_{h,i}}\sqrt{\int_{rotor} \sigma_u^2(x_i, y, z)\,dA} \tag{2}$$

where A is the area of the rotor, U and σ_u is the wind speed and turbulence in the wake region.

2.2 Mean velocity

To predict the mean flow field U in the multiple wake region, individual velocity deficits are combined based on the principle of Linear Superposition (LS) [2]:

$$U = U_0 - \sum_{i=1}^{n}(\Delta U_i) \tag{3}$$

$$\Delta U_i/U_{h,i} = F\left(C_{T,i}, I_{a,i}, x/D\right)\phi(r_i/\sigma_i) \tag{4}$$

where U_0 is the free stream wind speed, ΔU_i is the velocity deficit induced by each wind turbine calculated by the single wake model of Ishihara and Qian [5, 6], F and ϕ is the streamwise function and spanwise function, respectively, $C_{T,i}$ is the thrust coefficient of turbine i, σ_i is the representative wake width, r_i is the spanwise distance from the wake center, D is the rotor diameter.

2.3 Turbulence intensity

The individual added turbulences are then superimposed by the principle of Linear Superposition of Square (LSS) [3]:

$$\sigma_u^2 = \sigma_{u,0}^2 + \sum_{i=1}^{n}\left(\Delta\sigma_{u,i} + \Delta\sigma_{u,ij}\right)^2 \tag{5}$$

$$\Delta\sigma_{u,i}/U_{h,i} = G\left(C_{T,i}, I_{a,i}, x/D\right)\varphi(r_i/\sigma_i) \tag{6}$$

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where $\sigma_{u,0}$ is the free stream turbulence standard deviation, $\Delta\sigma_{u,i}$ is the added turbulence from each wind turbine based on the analytical Gaussian wake model [5, 6], G and φ is the streamwise function and spanwise function, respectively. Note that $\Delta\sigma_{u,ij}$ is a newly proposed correction term for turbine i to consider the wake mixing with the closet upstream turbine j as follows:

$$\Delta\sigma_{u,ij} = \frac{1}{2} k_{ij} \Delta\sigma_{u,i,tip} \tag{7}$$

$$k_{ij} = \begin{cases} \cos^2\left(\frac{\pi r_i}{D}\right) & A_1 \\ \sin^2\left(\frac{\pi(y-y_i)}{D}\right)\cos^2\left(\frac{\pi(z-H)}{D_{ij}}\right) & A_2 \\ 0 & else \end{cases} \tag{8}$$

where $\Delta\sigma_{u,i,tip}$ is the added turbulence at the tip side, A_1 and A_2 describe the turbulence correction areas for full and partial overlap, respectively.

2.4 Validation

Figure 1 and 2 show the horizontal contours and profiles of mean velocity and turbulence intensity at hub height for two wind turbines. It can be seen that for the full overlap wakes, both velocity deficits and turbulence become stronger, while in the partial overlap condition, the turbulence standard deviation in the overlap areas is weakened due to the wake mixing.

The quantitative comparison of velocity profiles show that the new proposed model based on the principle of LS with effective rotor onset turbulence presents favorable agreement with CFD results, while the conventional multi-zone model with Root Sum Square (RSS) superposition principle [1] generally underestimates deficits in the overlap region and overestimated them in the non-overlap areas. In addition, from the comparison of turbulence intensity profiles at selected positions, it can be found that the LSS superposition approach with the proposed turbulence correction for wake mixing well predicted the turbulence distribution in the wake region both for two turbines and six turbines. However, the IEC model

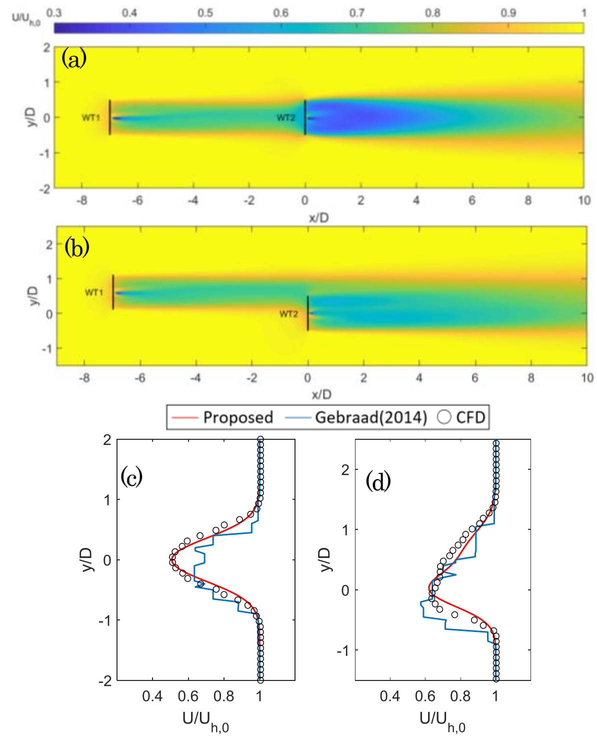

Figure 1. Contours and profiles (at x=4D) of mean velocity in the wake of two wind turbines: (a), (c) for full overlap; (b), (d), for partial overlap.

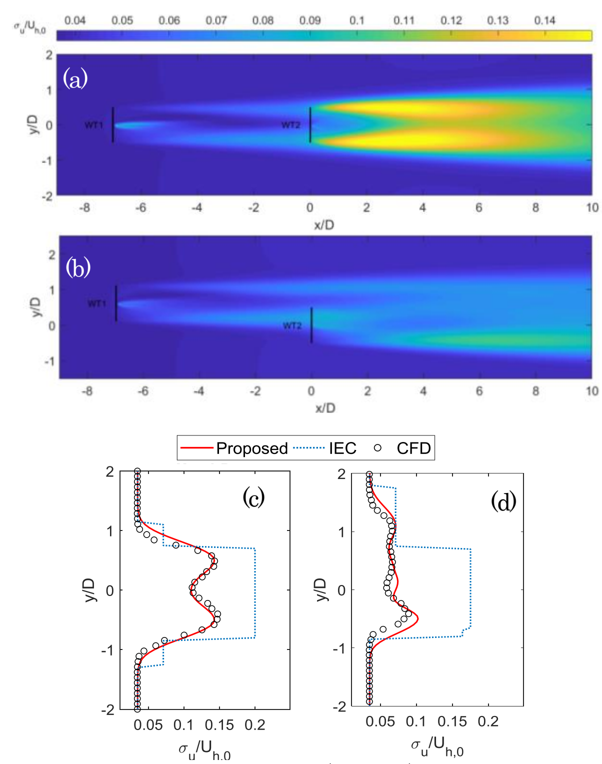

Figure 2. Contours and profiles (at x=4D) of turbulence intensity in the wake of two wind turbines: (a), (c) for full overlap; (b), (d), for partial overlap.

[3] generally gives conservative prediction in the near wake region for the full overlap wakes and large overestimation in the partial overlap wake regions.

3. Yaw-based Wind farm control

3.1 Optimization algorithm

This study focuses on the problem of optimizing AEP of a wind farm for a given site with an expected wind distribution, using the set-points for the yaw angels of the turbines as the optimization variable. Firstly, as formulated in Eq. (9), the optimization problem aims at finding the set of optimal yaw offset angles $\boldsymbol{\gamma}^{\text{opt}}(\theta, U) = \{\gamma_1^{\text{opt}}, \dots, \gamma_{N_T}^{\text{opt}}\}$ for N_T wind turbines, which maximizes the power output of the wind farm for the prescribed wind speed bin U_i and wind direction bin θ_j.

$$\boldsymbol{\gamma}^{opt}(U_i, \theta_j) = \underset{\boldsymbol{\gamma}}{argmax} \sum_{k=1}^{N_T} P_k(\gamma_1, \dots \gamma_{N_T}, U_i, \theta_j) \tag{9}$$

$$\text{Subject to } \gamma_{min} < \gamma_k < \gamma_{max}$$

Then the maximized AEP is calculated as the weighted sum of the total wind turbines' power production for each wind speed and direction bin, where the weighting is given by the frequency of each bin $f_{\theta,j}$ and $f_{U,i}$, times the number of hours in the year, N_h, as follows:

$$\max AEP = \sum_{i=1}^{N_{WS}} \sum_{j=1}^{N_{WD}} \left(\sum_{k=1}^{N} N_h P_k(\boldsymbol{\gamma}^{\mathbf{opt}}, U_i, \theta_j) \right) f_{\theta,j} f_{U,i} \tag{10}$$

To reliably handle optimization problem of larger numbers of design variables with fast converging, gradient-based optimization algorithm is adopted in this study.

3.2 Yaw offset limit

To determine the optimum yaw offset limit, a case study is performed for a test wind farm, which consists 25 NREL 5-MW wind turbines in 5 rows by 5 columns with the distance of 5D horizontally and vertically. The annual wind speed is assumed to be following the Rayleigh distribution (see Figure 3) and wind direction measurements at a near coastal site in Tomamae, Hokkaido of Japan is used as wind direction distribution (see Figure 4).

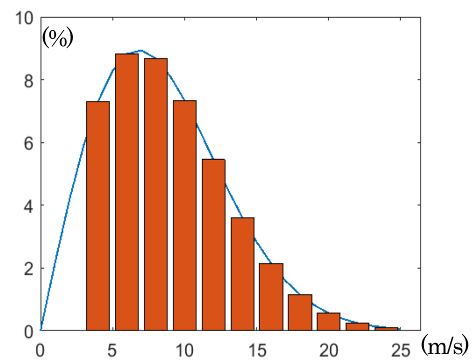

Figure 3. Rayleigh distribution with mean annual wind speed of 8.5m/s.

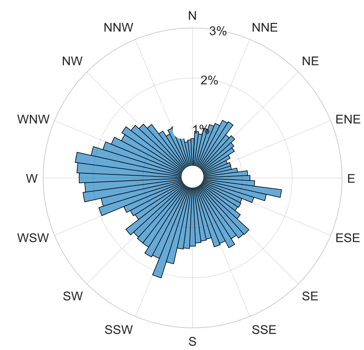

Figure 4. Wind direction distribution

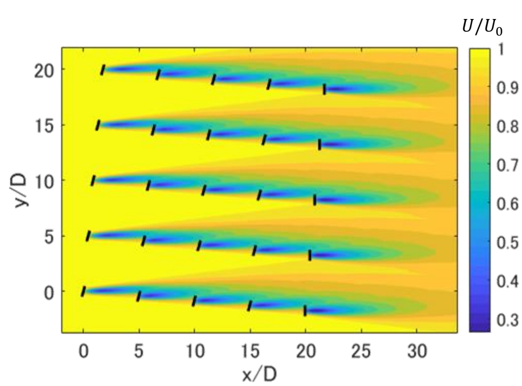

Figure 5. Example of optimized yaw offset and resulted wakes under certain wind speed and direction

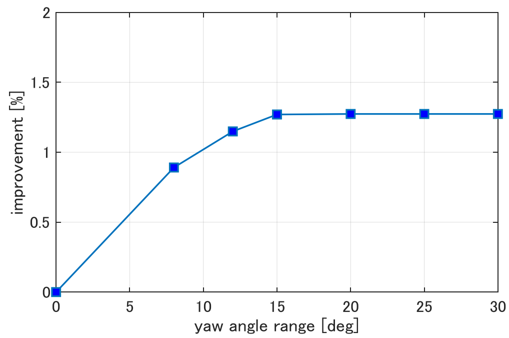

Figure 6. AEP improvements versus the yaw offset limit.

The yaw offset angles are optimized under each wind speed and direction bin by wake steering control, as shown in Figure 5. A series of yaw offset limit are tested and the obtained AEP improvements are shown in Figure 6, from which it can be concluded that the limit of ±15° can satisfy both the maximation of power and the safety requirement by IEC standard.

3.3 Effects of wind farm layout

To further investigate the effects of wind farm layout on the yaw-based wind farm control, three different wind turbine distances of 10D,7D and 5D (see Figure 7) are tested for the above mentioned test wind farm, where the other basic input parameters of wind condition are kept same as section 3.2. Generally, it can be concluded that wind farm with higher turbine density has higher potential of power gain by yaw-based wind farm control.

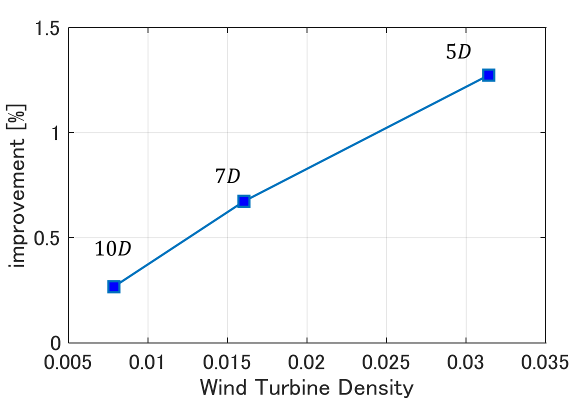

Figure 7. AEP improvements versus wind turbine density

4. Conclusions

(1) A new multiple wake model accounting for the local effective inflow on rotor is proposed, in which velocity deficits are combined by Linear Superposition, and turbulences are added using Linear Superposition of Square with a newly proposed correction term to consider the wake mixing. The proposed model is finally verified by comparison with numerical simulation results.

(2) A yaw-based wind farm power production maximization algorithm is developed based on the new multiple wake model and the yaw offset limit with ±15° is shown to be able to satisfy both the maximation of power production and the requirement of safety. Wind farm with higher turbine density has higher potential of power gain by yaw-based wind farm control.

References

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