Development of a New Wake Model Based on a Wind Tunnel Experiment

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Background

- Wake models are classified into two categories, analytical model and CFD based model
- Analytical models have advantages in the viewpoint of designing wind farm because of its simplicity and computational speed.
- Existing analytical wake models assume that the rate of wake recovery is constant.
- Due to this assumption, one single model cannot be used for both onshore and offshore wind farm.
- In Japan, a number of wind farms are constructed near coast, where ambient turbulence varies considerably depending on wind direction, indicating the need for universal wake model applicable to any ambient turbulence and thrust coefficient.

Object of this study

This study aims to develop a new universal wake model by taking the effect of turbulence on the rate of wake recovery into account.

References

[1] Gunner C. Larsen, Cookery Book for Wind Farm Load Calculations, Risø National Laboratory, Roskilde, Denmar
[2] I. Katic et al., A Simple Model for Cluster Efficiency, European Wind Energy Conference and Exhibition, Rome, Italy, 1986

Wind Tunnel Experiment

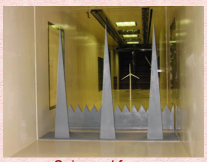
Wind Turbine Model

- A 1/100 scale model of Mitsubishi's MWT-1000 was settled in boundary layer wind tunnel.
- The height of the tower and the diameter of the rotor is 69.5cm and 57cm respectively.
- The motor inside the nacelle enables the rotor to rotate at realistic tip speed ratio.

Measurement

- Two types of flow with different ambient turbulence intensity (offshore and onshore) was simulated by spire and fence. For each flow two different experiment was carried out by changing the thrust coefficient of the turbine model.
- Vertical and horizontal velocity profiles were measured by split fiber probe at 2D, 4D, 6D and 8D behind the turbine.

Wind turbine model

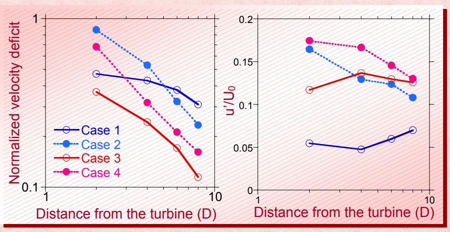


Spire and fence

	I_a=0.03 (offshore)	I_a=0.13 (onshore)
C_t=0.31	Case 1	Case 3
C_t=0.82	Case 2	Case 4

Experiment cases

Results



- When ambient turbulence corresponds to onshore flow (case 3 and 4), the rate of wake recovery is high due to the sufficient turbulence in the wake for any thrust coefficient.
- For the offshore case, however, when C_t is small (case 1), the lack of the turbulence in the wake makes the rate of recovery low. On the other hand when C_t is large (case 2), mechanical generated turbulence makes the wake recovery fast.

Analytical Model

Basic Idea

- Equation of momentum for axial symmetry flow

$$U_0\frac{\partial u_1}{\partial x} = \frac{\nu_t}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_1}{\partial r}\right)$$

Where, U_0 is wind speed of ambient flow at hub height, u_1 is velocity deficit ($u_1=U_0-u$) and ν_t is turbulence viscosity.

- Relation between drag force on the turbine and the loss of momentum flux.

$$\frac{1}{2}C_t\rho U_0^2\left(\frac{d}{2}\right)^2\pi = 2\pi\rho U_0\int_0^\infty u_1\cdot r\,dr$$

Where, d is diameter of the rotor and C_t is thrust coefficient of the turbine.

- Postulating the similarity of velocity profile, u_1 is expressed as

$$u_1 = CU_0x^{-p}\exp\left[-\left(\frac{r}{b(x)}\right)^2\right]$$

$$b(x) = 2\left(\frac{\nu_t}{U_0}\right)^{1-\frac{p}{2}}x^{\frac{p}{2}}$$

If exponent p is set to 1, these equations represent the solution of Schlichting. p shows the rate of wake recovery and the idea of proposed model is to regard p as a function of turbulence.

$$p = k_2(I_a+I_w)$$

Where, I_a is ambient turbulence and I_w is mechanical generated turbulence. I_w is modeled as

$$I_w = k_3\frac{C_t}{\max(I_a,0.03)}\left(1-\exp\left(-4\left(\frac{x}{10d}\right)^2\right)\right)$$

- Half the width at half depth $b_{1/2}(x)$ is assumed to be described as

$$b_{1/2}(x) = k_2C_t^{1/4}d^{1+\frac{p}{2}}x^{-\frac{p}{2}}$$

Summary

Based on the idea and assumption described above, following model is derived.

$$\frac{u_1(x,r)}{U_0} = \frac{C_t^{\frac{1}{2}}}{32}\left(\frac{1.666}{k_1}\right)^2\left(\frac{x}{d}\right)^{-p}\exp\left(-\frac{r^2}{b^2}\right)$$

$$b(x) = \frac{k_1C_t^{\frac{1}{4}}}{0.833}d^{1-\frac{p}{2}}x^{\frac{p}{2}}$$

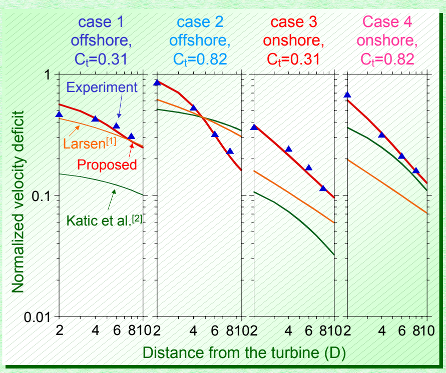
$$p = k_2(I_a+I_w)$$

$$I_w = k_3\frac{C_t}{\max(I_a,0.03)}\left(1-\exp\left(-4\left(\frac{x}{10d}\right)^2\right)\right)$$

$$\begin{cases} k_1 = 0.27 \\ k_2 = 6.0 \\ k_3 = 0.004 \end{cases}$$

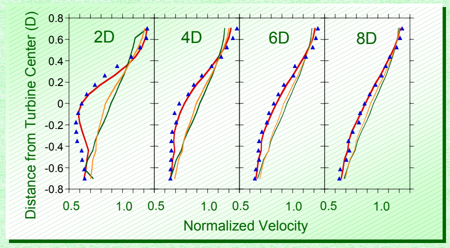
Verification

Center Line Velocity Deficit



- Proposed model shows good agreement with the experiment for all the cases.
- Larsen's model[1] underestimates the velocity deficit when applied to onshore case.
- Katic et al.'s model[2] shows fairy good agreement with the measurement for case 4, though for the other cases, it underestimates the velocity deficit.

Vertical Velocity Profile Downwind the Turbine



- Vertical velocity profiles is also well predicted by the proposed model.

Conclusion

Wind tunnel test with 1/100 scale turbine model was carried out and a new analytical wake model was proposed. Following results were obtained.

- Substantial velocity deficit is observed when thrust coefficient C_t is large and the rate of wake recovery is high when either ambient turbulence or mechanical generated turbulence is large.
- Conventional wake model designed for offshore wind farm underestimates the velocity deficit when applied to onshore wind farm and vice versa.
- A new wake model with the idea that the rate of wake recovery is a function of turbulence was proposed. The proposed model well predict the wake for any ambient turbulence and thrust coefficient