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A MODIFIED VON KARMAN MODEL FOR OFFSHORE WIND FIELD GENERATION

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ABSTRACT

Accurate modeling of three-dimensional turbulent flow field is necessary for dynamic analysis of wind turbines. In this study, a modified von Karman model is proposed for auto-correlations and auto-spectra as well as cross-correlations and cross-spectra for offshore wind generation. Measurement is carried out on an offshore meteorological mast for validation. Proposed model for auto-correlation and auto-spectra agree well with measurement data, while the original von Karman model underestimates auto-correlation and peak frequency in auto-spectra for lateral and vertical components. Proposed cross-correlation and cross-spectra also agree with measurements. The values of model parameters are proposed for ratios of standard deviation and normalized Reynolds stress based on measurement. Proposed values showed better agreement with measurement compared to the recommended values in IEC61400-1.

Keywords: spectrum, correlation, von Karman model, the IEC requirement

Introduction

The characteristics of input wind field has a considerable effect in dynamic response analysis of structures, and accurate modeling of three-dimensional turbulent flow field is necessary. A turbulence model can be written in both spectral and correlation form. Since the former is commonly used to describe turbulent characteristics and the latter can be directly used in generation of time series of wind field using AR model, it is meaningful for a turbulence model to be described in theoretically corresponding correlation and spectral form. Also the model parameters such as standard deviations and length scales are important factors of accuracy of the modeling. Therefore, for those structures like wind turbines and bridges that is built both onshore and offshore, it is necessary for the parameters to be newly proposed for offshore condition since the values provided in the design codes are usually based on onshore measurement.

Numerous turbulence models are proposed based on both theory and experiment in previous studies. Two representative models, Mann model (1988) and Kaimal model (1972), are commonly used for structure designing against wind. Mann model is a theoretical model based on von Karman model which is discussed below. However, due to its complex form of correlation, it is difficult to apply to generation of wind field. Kaimal model provides auto-spectrums for longitudinal, lateral, and vertical components based on measurements. However, since the model is empirical, no theoretical equations for auto-correlations are available. Similarly, one empirical model for uw component is provided by Kaimal for cross-correlation between different components at one point, although equations for cross-correlation are not available due to the same reason.

A well-known theoretical turbulence model is provided by von Karman (1948) where spectrums and corresponding correlation functions for each component are obtained theoretically assuming homogeneous and isotropic turbulence. It is commonly agreed that this model is compatible with measurement for longitudinal component, however Maeda and Makino (1992) pointed out that the spectrum of lateral and vertical components do not agree well with measurements of wind field which is normally non-isotropic.

In this study, a modified von Karman model with theoretically consistent correlations and spectra is proposed and the accuracy is validated by measurement from an offshore meteorological mast. The model parameters such as standard deviation ratio and length scale ratio for are proposed based on the measurement.

Proposed Turbulence Model

In the original von Karman model, auto-correlation of longitudinal component is described with a correlation function $f(r)$. Then the correlation function for lateral and vertical component $g(r)$ is obtained from $f(r)$ function using equation of continuity under isotropic turbulence field. However several measurements show that the shape of the auto-spectrum is quite similar for all the three turbulence components. Therefore, in this study, it is proposed to use the same shape for the auto-correlation and auto-spectrum for all the three components based on von Karman's longitudinal model. In other words, the use of same $f(r)$ function for all the components is proposed. Based on this idea, auto-correlation $\tilde{R}_i\,(U\tau)$ and auto-spectra $S_i(f)$ for i th component are written as Eq.(1) and Eq.(2).

$$\tilde{R}_i\,(U\tau) = \alpha_1 \left|a_1 U\tau\right|^{1/3} K_{1/3}(\left|a_1 U\tau\right|) \tag{1}$$

$$\frac{fS_i(f)}{\sigma_i^2} = \frac{4\left(fL_i/U\right)}{\left(1+70.8(fL_i/U)^2\right)^{5/6}} \tag{2}$$

where U is velocity of longitudinal component, σ_i is the standard deviation, L_i is the length scale, τ is the time lag and K_μ is modified Bessel function of second kind of the order of μ. The coefficient α_1 is $2^{2/3}/\Gamma(1/3) = 0.593$ and a_1 is $\sqrt{\pi}\Gamma(5/6)/\Gamma(1/3)L_i = 0.7468L_i$ respectively.

As auto-correlation is defined and described as $\tilde{R}_{ij}(U\tau) = E[u_i(t)u_i(t+U\tau)]/\sigma_i^2 = f(r)$, cross-correlation between different component defined as $\tilde{R}_{ij}(U\tau) = E[u_i(t)u_j(t+U\tau)]/\sigma_i\sigma_j$ is assumed to be able to be described with the same $f(r)$ function with a correcting factor α. This α can be determined as $\alpha = \overline{u_i u_j}/\sigma_i\sigma_j$ considering that $\tilde{R}_{ij}(0) = \overline{u_i u_j}/\sigma_i\sigma_j$. Once the Reynolds stress $\overline{u_i u_j}$ is obtained, the cross-correlation and cross-spectrum can be expressed with Eq.(3) and Eq.(4).

$$\tilde{R}_{ij}(U\tau) = \frac{\overline{u_i u_j}}{\sigma_i\sigma_j}\alpha_1 \left|a_1 U\tau\right|^{1/3} K_{1/3}(\left|a_1 U\tau\right|) \tag{3}$$

$$\frac{fC_{ij}(f)}{\sigma_i\sigma_j} = \frac{\overline{u_i u_j}}{\sigma_i\sigma_j}\frac{4\left(fL_{ij}/U\right)}{\left(1+70.8(fL_{ij}/U)^2\right)^{5/6}} \tag{4}$$

where $\tilde{R}_{ij}(U\tau)$ is the cross-correlation, $C_{ij}(f)$ is the cross-spectrum, $a_1 = 0.7468L_{ij}$ and length scale is assumed to be $L_{ij} = (L_i + L_j)/2$. When $i=1$ and $j=2$, the cross-correlation and the cross-spectrum are modeled as 0 following the assumption in atmospheric boundary layer.

In the following chapters, first these proposed models are validated with measurement data, and then the model parameters are proposed for offshore condition.

Outline of Measurement

Measurement data used for the validation of proposed model are from a meteorological mast located 3.1km offshore Choshi, Japan. The location and the outline of the met mast are shown in Figure 1. The mast is equipped with 3 sonic anemometers, 22 cup anemometers, 23 wind vanes, and sensors for other meteorological data such as pressure and temperature. In this study, data from the sonic anemometer located at 80m height is used, since proposed turbulence model focuses on wind speeds at one point. Data from other anemometers will be used in future discussion on spatial cross-correlation. Measurement has started from November 2012, and data from 16:00 to 16:40 on March 15th 2013 is used for the validation since high wind speed were observed for whole the day. Table 1 shows the mean wind speed, the mean wind direction, the standard deviation, and Reynolds stresses for each 10 minutes and the averaged data. Among the averaged values, only the length scales are not simple average of each 10 minutes data but are calculated with $\bar{L}_i = \bar{S}_i(0)/4\bar{\sigma}_i^2$ where $\bar{L}_i$ is average length scale, $\bar{S}_i(0)$ is average power spectrum at 0Hz, $\bar{\sigma}_i$ is averaged standard deviation. All the calculation is performed after removing linear trend in original time series, which will be discussed in the following section.

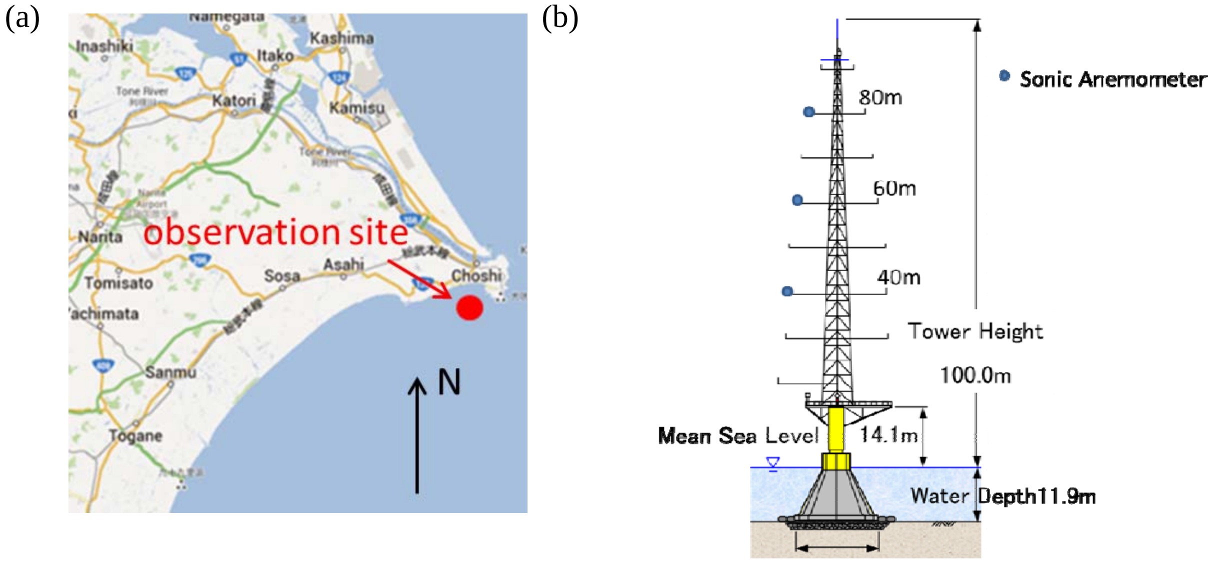

Figure.1 (a) Location and (b) detail of observation tower

Table.1 Statistical characteristics of measurement data

No.	Data time	U (m/s)	θ (deg)	σ_u (m/s)	σ_v (m/s)	σ_w (m/s)	$\overline{uv}$ (m^2/s^2)	$\overline{uw}$ (m^2/s^2)	$\overline{vw}$ (m^2/s^2)	L_u (m)	L_v (m)	L_w (m)
1	16:00 -16:10	17.7	206.5	1.15	0.89	0.73	-0.039	-0.16	-0.14	42.8	12.7	7.93
2	16:10 -16:20	19.2	206.3	1.09	0.94	0.75	0.043	-0.23	-0.18	51.7	12.3	8.19
3	16:20 -16:30	20.7	204.3	1.08	0.76	0.63	-0.042	-0.19	-0.16	56.4	20.4	12.4
4	16:30 -16:40	19.6	202.3	1.11	0.80	0.66	-0.11	-0.18	-0.24	51.1	26.3	16.8
Mean	-	19.3	204.9	1.11	0.84	0.69	-0.037	-0.20	-0.18	66.2	26.6	17.0

Data Analysis

As mentioned in the previous section, low frequency trend in 10 minutes time series are removed prior to any evaluations. The trend is assumed to be linear and is calculated using least-square method. Figure 2 (a) shows the length scales obtained from the original time series and it is seen that the values of longitudinal component and lateral component are not stable and some data are several times larger than other cases. Figure 2 (b) shows the length scales calculated after removing linear trend. The effect of detrending can be seen in that values for longitudinal and lateral components became almost same for every case while those for vertical component did not change significantly. For calculation of power spectra, 10 minutes data are divided into blocks of 2048 samples and are adapted to discrete fast Fourier transformation. The spectra are then applied to Hamming window before averaged. Finally calculated correlations and spectra for data No.1 to No.4 are averaged and bin average is adapted to the spectra at the size of 0.025Hz.

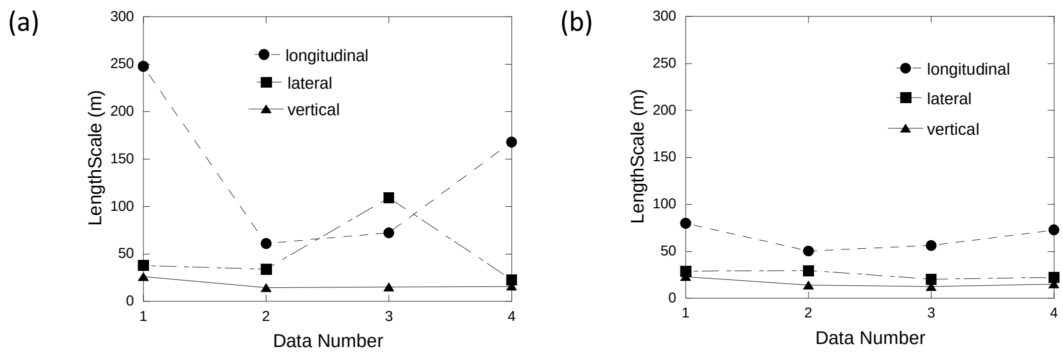

Figure 2. Length scales for three components (a) before and (b) after removing trend

Validation of Proposed Model

Figure 3 shows the comparison between proposed and measured auto-correlation and normalized spectrum for three components. For longitudinal component, where proposed model is the same as von Karman model, the measurement shows good agreement with the models. For the lateral and vertical components, where the proposed model differs from von Karman's original model, there is a small difference between proposed and original von Karman model for the lateral component and a clear difference for the vertical component. In both frequency and time domain, the proposed model gives satisfying results when compared to the measurements.

Comparison of proposed and measured cross-correlation and cross-spectrum between different components are shown in Figure 4. Figure 4 (a) shows that there is no clear correlation between longitudinal and lateral components, which agrees with the assumption. On the other hand, clear correlation exists between longitudinal and vertical (b) and lateral and vertical (c) components. The proposed cross-correlation models agree well with the measurement data.

Model Parameters of Proposed Model

In the proposed model, the mean wind speed U, the standard deviation σ_i, the length scale L_i and normalized Reynolds stress $\overline{u_i u_j} / \sigma_i \sigma_j$ are the parameters that have to be determined. Since U, σ_1 and L_1 are usually determined by design requirement and the ratio of length scales are calculated from the ratio of standard deviation, the ratios of standard deviations and normalized Reynolds stresses are the parameters that have to be specified when no measurement data is available. For calculation of the ratio of length scale, auto-spectrum in asymptotic inertia subrange S_i is derived from Eq.(2) as Eq.(5). Then, taking the ratio of Eq.(5) between two components, Eq.(6) is obtained where the ratio of spectra is $S_1 / S_2 = S_1 / S_3 = 0.75$ considering the theory of locally isotropic turbulence in inertia subrange,

In order to compare the values of parameters with the design code, the international design code of wind turbines, IEC61400-1 is discussed below as an example. In IEC61400-1, the ratios of standard deviations are determined as $\sigma_2 = 0.8\sigma_1$ and $\sigma_3 = 0.5\sigma_1$ for Kaimal model whereas the measured result is and $\sigma_2 = 0.76\sigma_1$ and $\sigma_3 = 0.62\sigma_1$ which agree with the value $\sigma_2 = 0.82\sigma_1$ and $\sigma_3 = 0.63\sigma_1$ proposed by Moraes (1988) based on the Kansas observation. Therefore the parameters $\sigma_2 = 0.8\sigma_1$ and $\sigma_3 = 0.6\sigma_1$ are proposed in this study for offshore condition. The predicted ratio of length scale for lateral component L_2 / L_1 is 0.33, which is same as that in Kaimal model and close to the measurement of 0.40. The predicted ratio of length scale for vertical component L_3 / L_1 is 0.081, which shows some difference compared with measured ratio of length scale $L_3 / L_1 = 0.14$ and is better than Kaimal model with $L_3 / L_1 = 0.081$. Proposed values for Reynolds stress are determined from observation as $\overline{u_1u_2} / \sigma_1\sigma_2 = 0$, $\overline{u_1u_3} / \sigma_1\sigma_3 = -0.13$, and $\overline{u_2u_3} / \sigma_2\sigma_3 = 0.14$. Comparison of model parameters of observation, Kaimal model and proposed modified von Karman model are shown in Table 2.

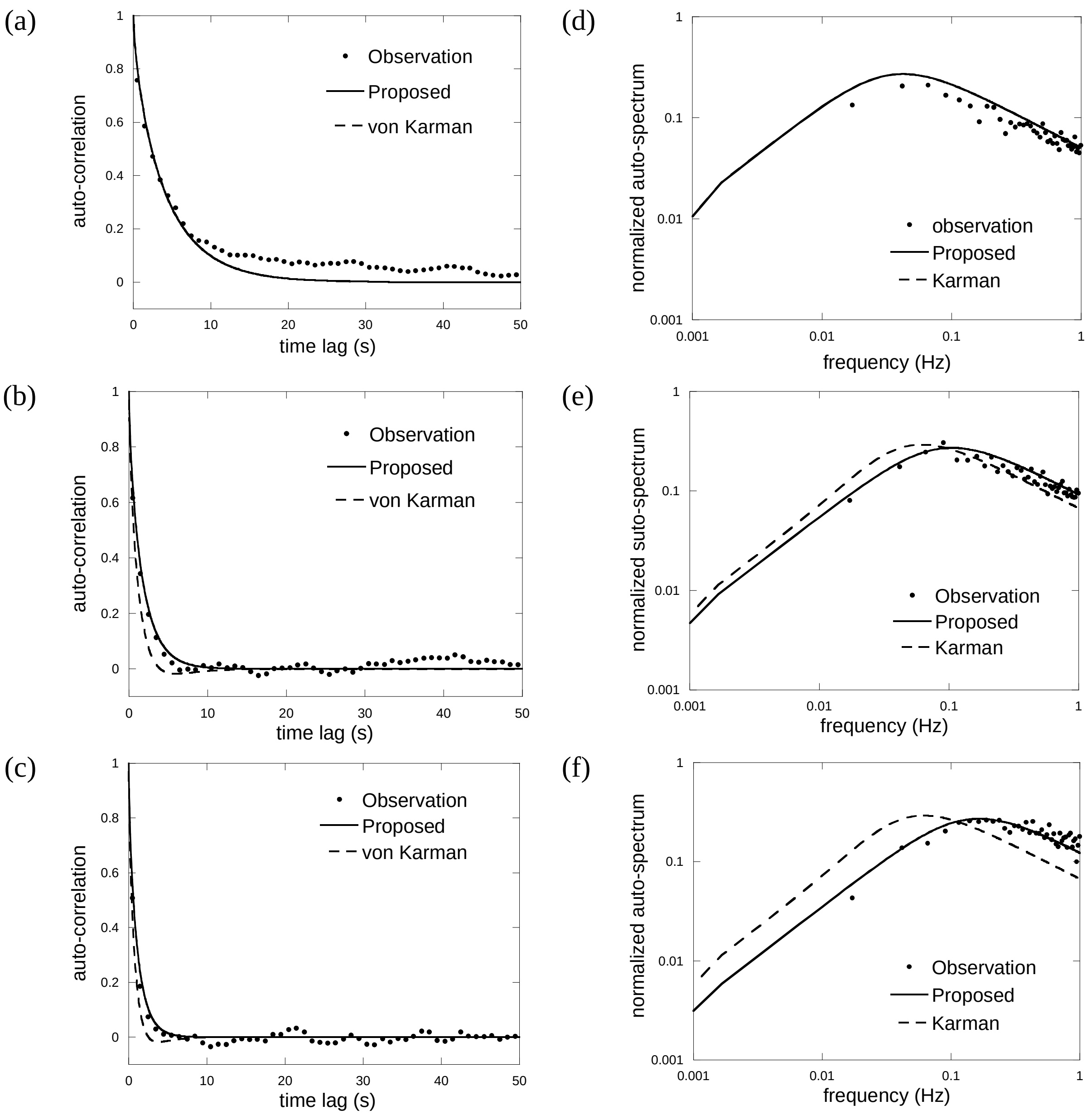

Figure 3 Comparison between calculated and measured auto-correlation and normalized auto-spectrum for (a) (d) longitudinal, (b) (e) lateral and (c) (f) vertical component

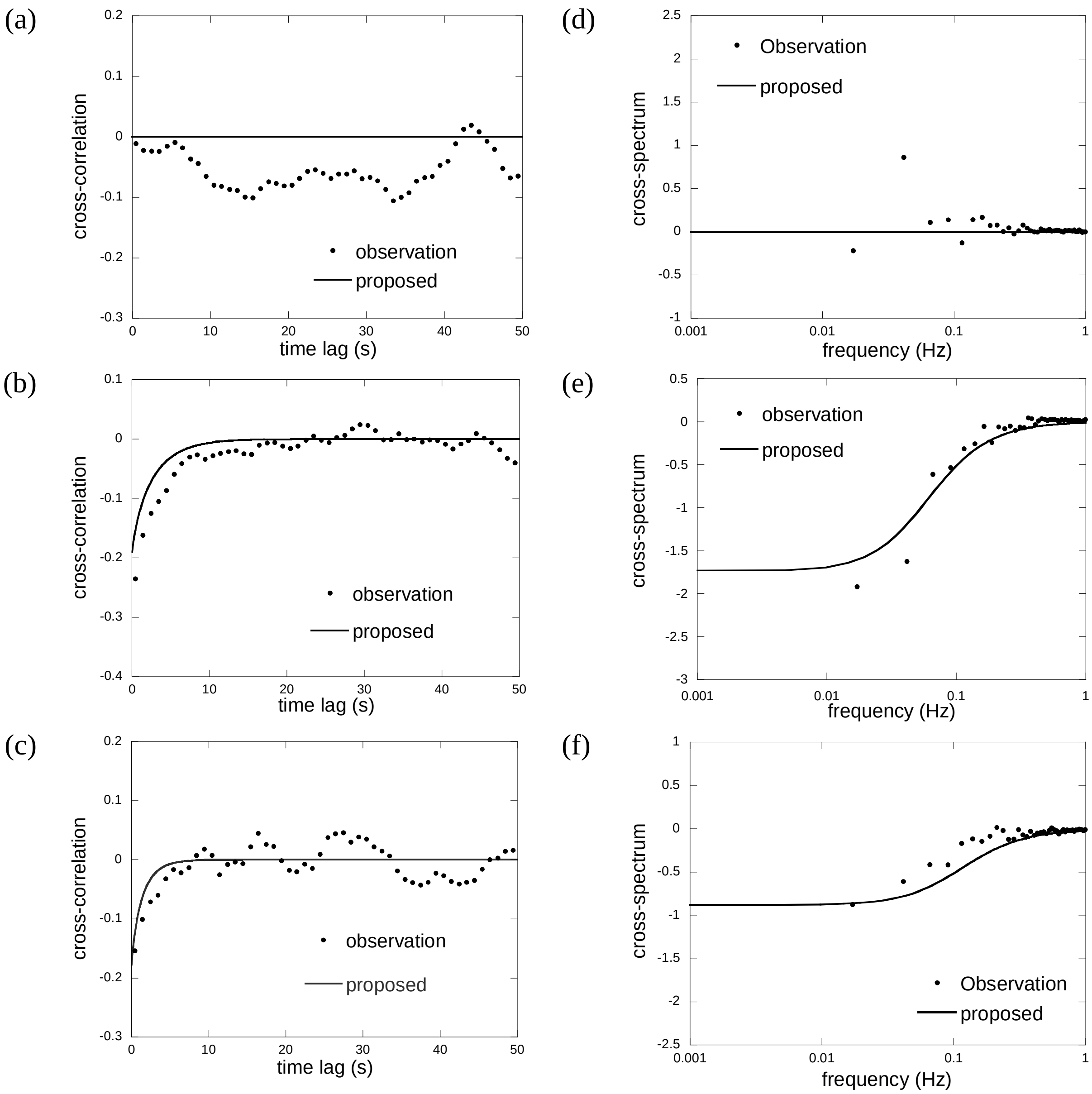

Figure 4 Comparison between calculated and measured cross-correlation and cross-spectrum for (a) (d) longitudinal and lateral, (b) (e) longitudinal and vertical and (c) (f) lateral and vertical component

$$\frac{fS_i(f)}{\sigma_i^2} \rightarrow \frac{4\left(\frac{fL_i}{U}\right)}{70.8^{5/6}\left(\frac{fL_i}{U}\right)^{5/3}} = 0.115\left(\frac{fL_i}{U}\right)^{-2/3} \tag{5}$$

$$\frac{\dfrac{fS_i(f)}{\sigma_i^2}}{\dfrac{fS_j(f)}{\sigma_j^2}} = \frac{0.115\left(fL_i/U\right)^{-2/3}}{0.115\left(fL_j/U\right)^{-2/3}} \rightarrow \frac{L_i}{L_j} = \left(\frac{S_j}{S_i}\right)^{3/2}\left(\frac{\sigma_i}{\sigma_j}\right)^{3} \tag{6}$$

Table 2. Parameters in Kaimal and proposed model

	Observation	IEC(Kaimal)	Proposed
σ_2/σ_1	0.76	0.8	0.8
σ_3/σ_1	0.62	0.5	0.6
L_2/L_1	0.40	0.33	0.33
L_3/L_1	0.26	0.081	0.14
$\overline{u_1u_2}/\sigma_1\sigma_2$	-0.028	-	0
$\overline{u_1u_3}/\sigma_1\sigma_3$	-0.124	-	-0.13
$\overline{u_2u_3}/\sigma_2\sigma_3$	-0.148	-	-0.15

Conclusions

In this study, a modified von Karman model is proposed to describe the non-isotropic wind field in the forms of theoretically consistent correlation and spectrum. Following conclusions are obtained:

1) Proposed auto-correlation and auto-spectrum model agree well with measurement data for all components, while the original von Karman model under-estimates the value of auto-correlation and the peak frequency in auto-spectrum for lateral and vertical component.
2) Proposed cross-correlation and cross-spectrum also agree well with measurements for longitudinal and vertical component, and lateral and vertical component. The longitudinal and lateral component can be assumed as 0 for both correlation and spectrum .
3) Ratios of standard deviation and normalized Reynolds stress in the modified von Karman model are proposed for offshore condition. Proposed values show better agreement with measurement than those in Kaimal model of IEC61400-1.

Appendix

The original von Karman model for auto-correlations and spectra shown in Figure 3 are calculated with Eq.(a1) and E.(a2) for longitudinal component, and Eq.(a3) and Eq.(a4) for lateral and vertical component.

$$\tilde{R}_i\,(U\tau) = \alpha_1 \left|a_1 U\tau\right|^{1/3} K_{1/3}(\left|a_1 U\tau\right|) \tag{a1}$$

$$\frac{fS_i(f)}{\sigma_i^2} = \frac{4\left(fL_i/U\right)}{\left(1+70.8(fL_i/U)^2\right)^{5/6}} \tag{a2}$$

$$\tilde{R}_i\,(U\tau) = \alpha_1 \left|a_1 U\tau\right|^{1/3} (K_{1/3}(\left|a_1 U\tau\right|) - \frac{\left|a_1 r\right|}{2} K_{2/3}(\left|a_1 r\right|)) \tag{a3}$$

$$\frac{fS_i(f)}{\sigma_i^2} = \frac{2(L_i/U)(1+188.8\left(fL_i/U\right)^2)}{\left(1+70.8(fL_i/U)^2\right)^{11/6}} \tag{a4}$$

The auto-spectrum for Kaimal model is shown in Eq.(a5) for the three components, and the empirical spectrum model for longitudinal and vertical components is shown in Eq.(a6).

$$\frac{fS_i(f)}{\sigma_i^2} = \frac{4\left(fL_i/U\right)}{\left(1+6(fL_i/U)\right)^{5/3}} \tag{a5}$$

$$\frac{fC_{uw}(f)}{\sigma_u\sigma_w} = \frac{1.066\left(fL_i/U\right)}{\left(1+1.745(fL_i/U)\right)^{7/3}} \tag{a6}$$

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