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An attempt at estimating the instantaneous velocity field of a nonstationary flow from velocity measurement at a few points

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Abstract. Generally speaking, velocity measurement devices such as hot-wire and LDV (laser Doppler velocimeter), which may be called Eulerian type flow measurement devices, can record temporally continuous and precise velocity variations. If the flow is laminar, steady or unsteady, or in a transition stage to turbulence, repeated measurements will yield the whole image of the flow. However, even with an array of these devices, the image of the instantaneous flow field cannot be obtained, if the flow is turbulent. Usually, to detect the structure of turbulence conditional ensemble averages are taken. Because of the arbitrariness of the conditions imposed, the result may be different from the true ones. In this paper, it is attempted to reconstruct the instantaneous image of the whole flow field from velocities detected instantaneously at sparse points. The first step is the spatial interpolation of the velocity field by the method of "virtual load" proposed by Hino. The second step is to regulate them by the mass conservation technique which is used in the field of meteorology. The usefulness of the method has been checked by numerical simulations. An example of analysis on the instantaneous feature of a large eddy structure is given.

1. Introduction

In the field of meteorology, a method of estimating the whole wind field from velocity data at a limited number of observation points was first proposed by Sasaki (1958) and further extended and tested by others, for instance Sherman (1978), Ichikawa and Shikata (1982), and Kitada et al. (1981). However, such a method has never been applied, to the authors' knowledge, in the case of laboratory experiments. This may be due to the fact that detailed data on the whole flow field are obtainable, even if the flow is nonstationary, by repeating the same experiment under the same conditions. For instance, the precise process of nonlinear transition from laminar flow to turbulence may be obtained by repeating the velocity data sampling again and again, using a detection probe to ascertain the generation and passage of the identical flow pattern (e.g., Nishioka et al., 1981).

In the two-dimensional cases, flow field may be reconstructed from the continuity equation by assuming Taylor's hypothesis of frozen velocity field, if velocity profiles of one component on a cross-section are obtained simultaneously and continuously, for instance by a rake of hotwires or SLV (scan-type laser Doppler velocimeter; Hino, Nadaoka, Kobayashi, Hironaga and Muramoto, 1986).

In the case of the elementary processes of turbulent motion such as bursting, recourse has been taken to the technique of conditional sampling to deduce the turbulence transport mechanism, yielding the model named the "banana vortex" by Fukunishi and Sato (1987). Although the technique of conditional sampling and the data processing of ensemble averages greatly advanced our understanding of the elementary process of turbulent motion, the result is nevertheless the averaged image and not the instantaneous one (Hino et al., 1983). If we narrow the conditions of the sampling, the result will be a strictly conditioned and thus rather limited or distorted image of the true features. This is the reason why we are trying to obtain the instantaneous picture of a turbulent flow field.

Recently, the technique of flow visualization has developed remarkably. However, the method cannot escape from the defect that the pictures obtained by the flow visualization technique are the integrated result of a fluid motion. To mention an example of misunderstanding of the result, an image of a vortex does not necessarily mean that there exists a vortical motion; it may be rather a corpse, or a vestige of the vortical motion that existed beforehand. In other words, the flow visualization technique might give an erroneous picture of fluid motion if it is not properly and cautiously interpreted.

2. Estimation of velocity field based on the mass-conservation principle

2.1. Initial guess of velocity field by the "virtual load" method

The method of velocity field estimation described in this paper is based on the mass conservation principle (Sasaki, 1958; Sherman, 1978; Ichikawa and Shikata, 1982; Kitada et al., 1981; Hino, Meng and Murayama, 1989, 1990). As a first stage, a rough estimation of a whole flow field by the inter- and extrapolation of measured data must be performed. The writers apply the " virtual load" method proposed by Hino (1975). The method is to interpolate some quantity of given data (velocity component, effluent concentration, rainfall intensity, etc.) as the deflection of a hypothetical elastic plate (of length a, width b, thickness t and elasticity E) by a few virtual loads concentrated at some arbitrarily chosen points (ξ, η). In our case, the plate deflection corresponds to the velocity component u or v. The method has been applied not only to drawing contour maps of atmospheric pollution or rainfall intensity (Hino, Yoshikawa, and Kurihara, 1977; Hino, 1985), but also to solve the partial differential equation (Hino and Miyanaga, 1975).

If the points of virtual loading are chosen properly, the problem is reduced to that of determining the value of the optimal virtual loads so as to coincide the deflections of the hypothetical elastic plate with the value of given data (i.e. by the collocation method). The deflection of the elastic plate U_i $(=u_i$ or $v_i)$ at a point (x_i, y_i) is represented in terms of the Green function $G(x, y; \xi, \eta)$ as

$$U(x_i, y_i) = \sum_j G(x_i, y_i; \xi_j, \eta_j) P_j, \tag{1}$$

where the Green function $G(x_i, y_i; \xi_j, \eta_j)$ describes the plate deflection at a point (x_i, y_i) by a concentrated unit load acting on the point (ξ_j, η_j), P_j is the virtual load at a point (ξ_j, η_j), and the indices i $(=1, 2, \ldots, M)$ and j $(=1, 2, \ldots, N)$ denote, respectively, the number of detecting points of deflection and the number of loading points. The Green function for a rectangular elastic plate, which freely supported at four edges, is given by

$$G(x, y; \xi, \eta) = \frac{4}{\pi^4 abD} \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} \frac{\sin(k\pi x/a)\sin(l\pi y/b)\sin(k\pi\xi/a)\sin(l\pi\eta/b)}{k^2/a^2 + l^2/b^2}, \tag{2}$$

where $D = Et^3/12(1-\nu^2)$ and ν is the Poisson ratio. The linear simultaneous equations for the unknown P_j, eq. (1), are solved, for instance, by the Gauss–Jordin method.

2.2. Improvement of initial rough estimate by MASCON model

In order to improve the initial rough estimate of the velocity field, the MASCON model (Sasaki, 1958) is applied. The initial estimate of the velocity field by the method described above is modified to satisfy the continuity equation by the variational technique with the additional criteria to minimize

$$\alpha_1^2(u-u_0)^2+\alpha_1^2(v-v_0)^2+\alpha_2^2(w-w_0)^2, \tag{3}$$

where α_1 and α_2 are the weights.

Introducing the Lagrange multiplier λ, the following functional F^* should be minimized:

$$F^*=\int_V\left[\alpha_1^2(u-u_0)^2+\alpha_1^2(v-v_0)^2+\alpha_2^2(w-w_0)^2 +\lambda\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)\right]\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z, \tag{4}$$

where V is the flow domain. By putting the first variation of eq. (4) equal to zero, the values of u, v, and w are expressed as

$$u=u_0+\frac{1}{2\alpha_1^2}\frac{\partial\lambda}{\partial x}, \tag{5a}$$

$$v=v_0+\frac{1}{2\alpha_1^2}\frac{\partial\lambda}{\partial y}, \tag{5b}$$

$$w=w_0+\frac{1}{2\alpha_2^2}\frac{\partial\lambda}{\partial z}. \tag{5c}$$

Substituting the above equations into the continuity equation,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{6}$$

the following partial differential equation of elliptic type is derived (Sasaki, 1958),

$$\frac{\partial^2\lambda}{\partial x^2}+\frac{\partial^2\lambda}{\partial y^2}+\left(\frac{\alpha_1}{\alpha_2}\right)^2\frac{\partial^2\lambda}{\partial z^2}=-2\alpha_1^2\left(\frac{\partial u_0}{\partial x}+\frac{\partial v_0}{\partial y}+\frac{\partial w_0}{\partial z}\right). \tag{7}$$

The value of λ is obtained by solving eq. (7) by the SOR (successive overrelaxation) method. In solving eq. (7), the boundary conditions for λ are given as $\lambda=0$ when the fluid is allowed to pass freely through the boundaries. If the velocity is zero at the boundary, the condition $\partial\lambda/\partial n=0$ is adopted. Since the process of deriving eq. (7) is well explained by Sasaki (1958) and the original idea is his, we refer to his paper rather than giving the reasoning in detail.

The spatial resolution is of the order of the separation distances between neighboring probes, and the area of reliable estimation is that covered by the measuring probes.

3. Results of simulation experiment

3.1. Two-dimensional case

The applicability of the above method has been tested by simulation experiments. Since one of our aims is to detect the instantaneous image of the vortical motion of the so-called "banana

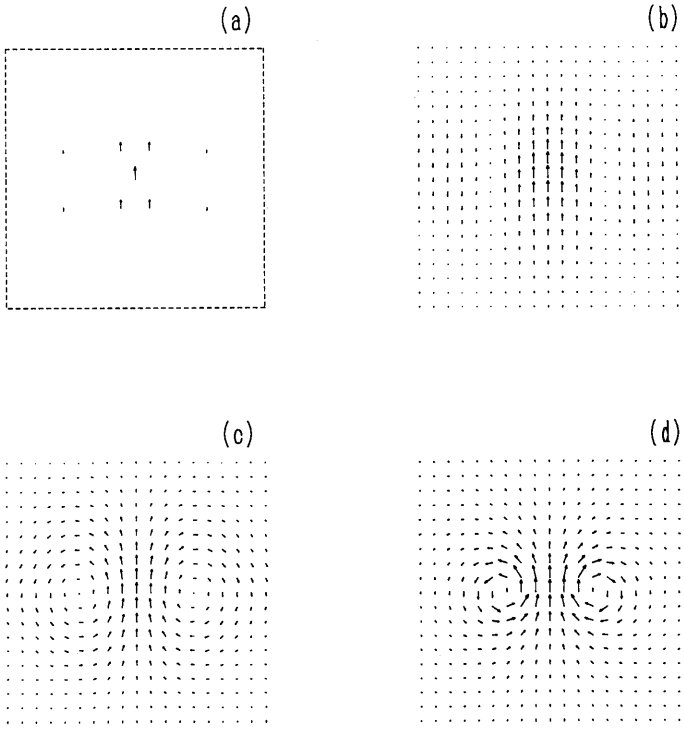

Fig. 1. Test of the applicability of the present method to a two-dimensional case, when only one velocity component w is given at a few points. (a) Velocity component w given at nine points. (b) Velocity field estimated roughly by the "virtual method". (c) Velocity field corrected by the "MASCON" model. (d) True velocity field of a pair of Rankine's vortices.

eddies" in the elementary process of turbulence, the true flow field is chosen as a pair of the Rankine vortices given by

$$u_0 = -\frac{rk}{r_0} \quad (0 \leqslant r \leqslant r_0),$$
$$= -\frac{k}{r} \quad (r_0 < r), \tag{8}$$

where the intensity of a vortex is given as $k = 3$.

3.1.1. Estimation of velocity field from velocities of one component (w) at a few points

At first, the estimation method has been examined under the strict condition that only one velocity component in the (x, z) domain is measurable. A rather lucky case, in which the net of measuring points detected the central part of the vortices, has been tested. At a few points, velocities of only one component (w) are measured, as shown in fig. 1a. A first guess as to the velocity field made by the "virtual load" method is as shown in fig. 1b. Next, the whole velocity field of u and w is estimated using the MASCON model to satisfy the continuity equation, the first guess of u being set at $u = 0$, and the weighting factors being $\alpha_1 = 1$ and $\alpha_2 = 3$. The result shown in fig. 1c compares relatively well with the true velocity field of fig. 1d.

3.1.2. Estimation of velocity field from velocities of two components (u and w) at a few points

As a second step, the velocities of two components (u and w), at a few points which are at the same locations as those in fig. 1a are given as shown in fig. 2a. The first guess by the

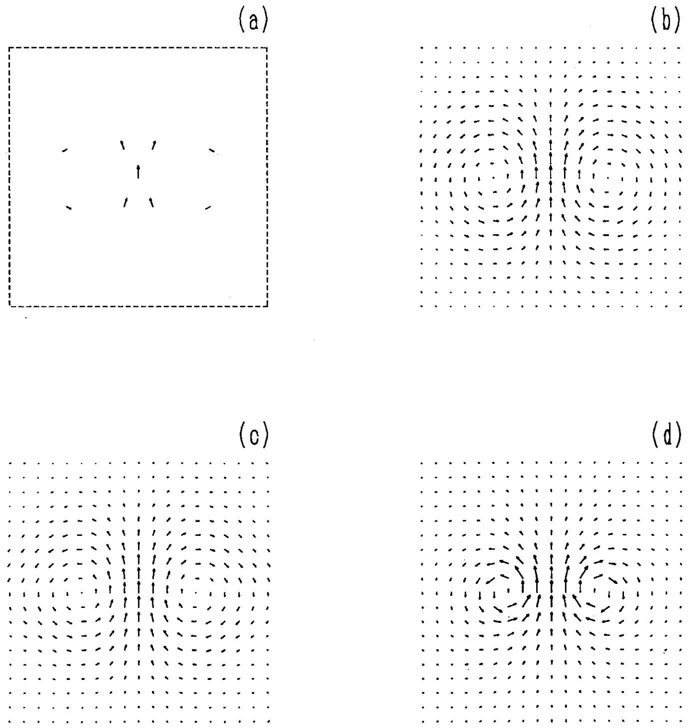

Fig. 2. Test of the applicability of the present method to a two-dimensional case, when two velocity components (u, w) are given at a few points. (a) Velocity components, u and w, given at nine points. (b) Velocity field estimated roughly by the "virtual method". (c) Velocity field corrected by the "MASCON" model. (d) True velocity field of a pair of Rankine's vortices.

"virtual load" method and the improved one by the MASCON model are illustrated in figs. 2b and 2c, respectively.

3.2. Estimation of three-dimensional flow field from a point source

A flow from a point source (fig. 3a) has been selected as a simple test case of the applicability of the MASCON model to three-dimensional cases. Fig. 3b shows the velocity vectors (v, w) on the YZ plane at $x = 3.9$. The data for u and v velocity components on a cross-section, putting $w = 0$, are given only at 10 parallel planes as shown in fig. 3c. The abovementioned method has been applied with a high degree of success, as shown in fig. 3d, which compares remarkably well with the real velocity shown in fig. 3b.

4. Preliminary results on instantaneous image of bursting

With the conventional technique of conditional sampling and ensemble averages, we can only know the mean and smoothed features of various sized individual eddies in turbulent motions. Although the conditional ensemble average naturally gives, as a result of the averaging process, a symmetric shape as an element of turbulent motion, such as a "banana vortex pair", it is possible that it is asymmetric, for instance, with a bigger and stronger vorticity on one leg of the pair vortices.

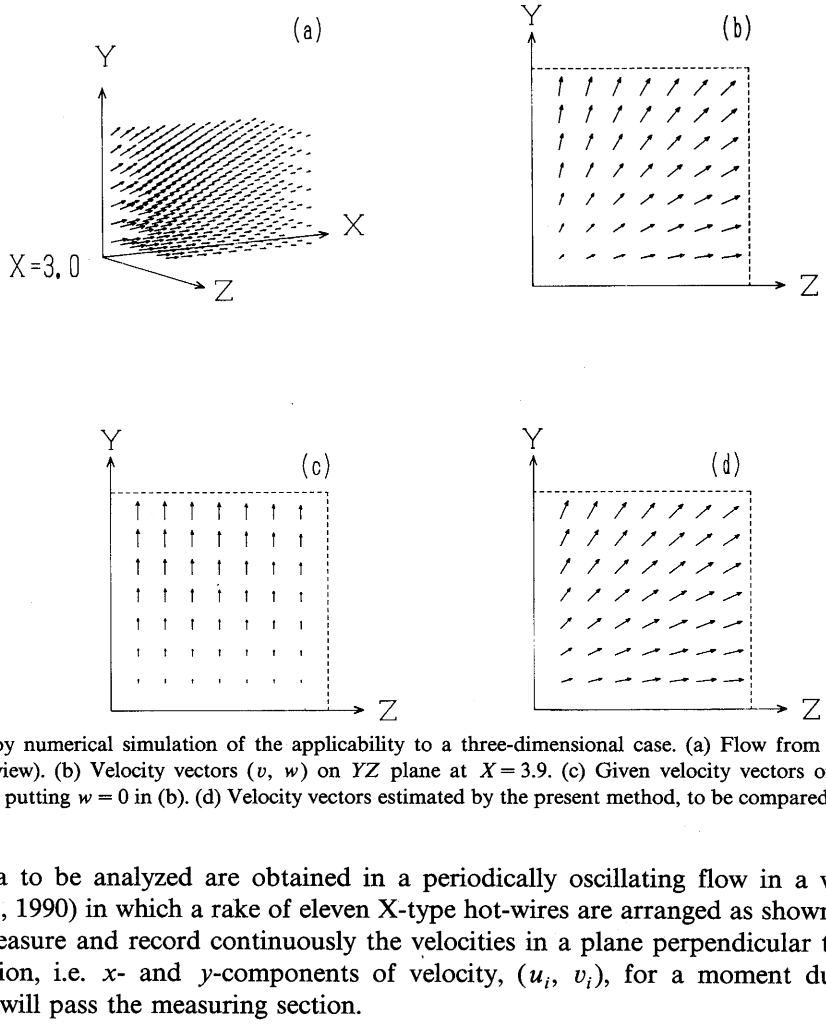

Fig. 3. Test by numerical simulation of the applicability to a three-dimensional case. (a) Flow from a point source (perspective view). (b) Velocity vectors (v, w) on YZ plane at $X = 3.9$. (c) Given velocity vectors on YZ plane at $X = 3.9$, putting $w = 0$ in (b). (d) Velocity vectors estimated by the present method, to be compared with (b).

The data to be analyzed are obtained in a periodically oscillating flow in a wind tunnel (Hino et al., 1990) in which a rake of eleven X-type hot-wires are arranged as shown in fig. 4 in order to measure and record continuously the velocities in a plane perpendicular to the mean flow direction, i.e. x- and y-components of velocity, (u_i, v_i), for a moment during which turbulence will pass the measuring section.

Fig. 5a shows the y-velocity components measured at the 11 points indicated in fig. 4. Fig. 5b illustrates the initial rough estimates of the instantaneous velocities at the section of the probe arrangement, obtained by the "virtual load" method. From a such successive measure-

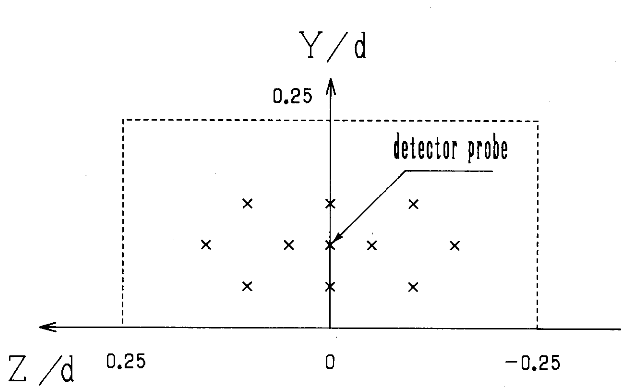

Fig. 4. Arrangement of eleven X-type hot-wires in a plane perpendicular to the flow axis, to record the u, v components.

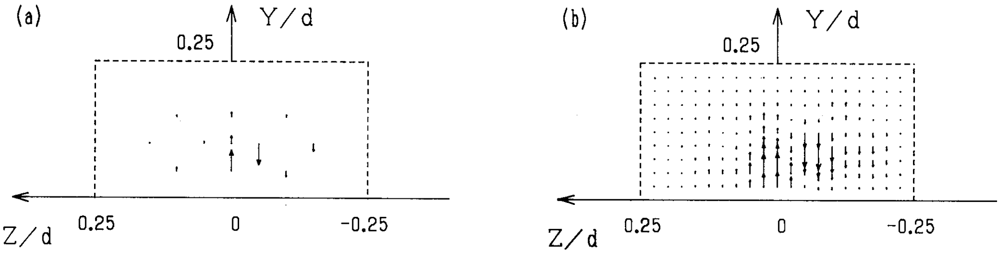

Fig. 5. (a) The y-velocity components measured at 11 points indicated in fig. 4. (b) The initial rough estimates of the instantaneous velocities in an oscillatory turbulent flow by the "virtual load" method.

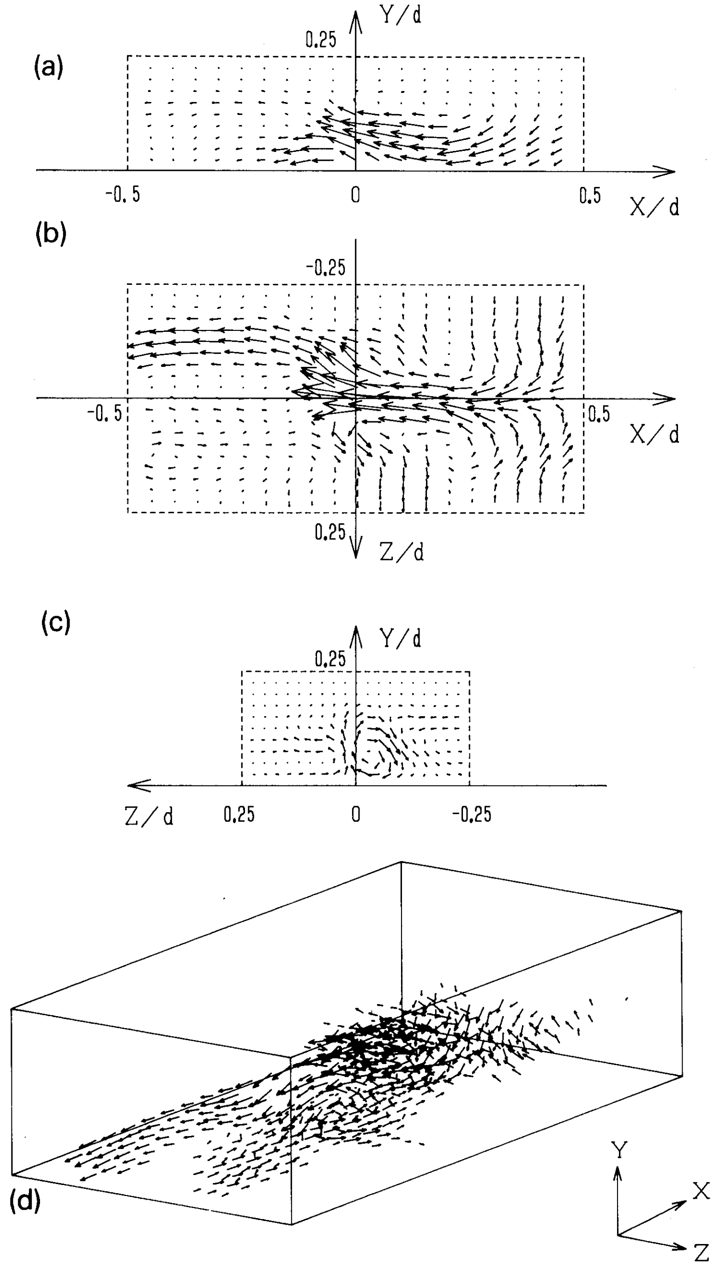

Fig. 6. (a–c) The instantaneous image of a turbulence field of the oscillatory flow, obtained based on Taylor's frozen turbulence hypothesis. (d) A bird's-eye view of the instantaneous turbulent motion in an oscillatory turbulent flow.

ment of u and v at the measuring section, the instantaneous image of three-dimensional turbulence field has been derived, applying Taylor's "frozen turbulence" hypothesis, as shown in figs. 6a–6c. Fig. 6d is a bird's-eye view of the instantaneous turbulent motion.

A strong ejection of a low-speed fluid is seen from figs. 6a and 6b. Fig. 6c, which is a vertical cross-section of the low-speed ejection motion, indicates that the motion is caused by a vortical motion with a downstream (longitudinal) axis perpendicular to the (y, z)-plane. Detailed analysis and discussion of further experimental data will be described in a subsequent paper.

5. Conclusion

Estimation of an instantaneous velocity field, 2- or 3-dimensional, from velocity measurement at a few points has been attempted by applying the MASCON model together with the "virtual load" method of inter- and extrapolation. The applicability of the method has been checked by simulation experiments.

A preliminary experiment on the instantaneous image of turbulence in an oscillatory flow captured the instantaneous picture of the ejection motion accompanied by a vortical motion.

References

Fukunishi, Y. and H. Sato (1987) Formation of intermittent region by coherent motions in the turbulent boundary layer, *Fluid Dyn. Res. vol. 2,* 113–124.

Hino, M. (1975) Proposal and explanation of "virtual method" as a means of approximation, *Tech. Report No. 18, 89–96,* Department of Civil Engineering, Tokyo Institute of Technology (in Japanese).

Hino, M. (1985) Short-term rainfall prediction by the "virtual load" method, *Proc. 29th Japanese Conf. on Hydraulic Research JSCE,* pp. 203–208 (in Japanese).

Hino, M., M. Kashiwayanagi, A. Nakayama and T. Hara (1983) Experiments on the turbulence statistics and the structures of a reciprocating oscillatory flow, *J. Fluid Mech. 131,* 363–400.

Hino, M., Y. Meng and M. Murayama (1989) An attempt to estimate an instantaneous velocity field, *Tech. Report No. 41,* Department of Civil Engineering, Tokyo Institute of Technology (in Japanese with English abstract) 1–7.

Hino, M., Y. Meng and M. Murayama (1990) Measurement of an instantaneous image of three-dimensional ordered turbulence structure in an oscillatory flow, *Preprint, 45th Annual Conf. of JSCE, Part II* (in Japanese) pp. 388–389.

Hino, M. and Y. Miyanaga (1975) Wave force and wave scattering by Green's function method and imaginary plate-load approximation, *Proc. JSCE 237,* 51–62 (in Japanese).

Hino, M. K. Nadaoka, T. Kobayashi, K. Hironaga and T. Muramoto (1986) Flow structure measurement by beam scan type LDV, *Fluid Dyn. Res. 177–190.*

Hino, M., S. Yoshikawa and T. Kurihara (1977) Experience in air pollution prediction by statistical and stochastic prediction technique, *Proc. JSCE 268*, 47–62 (in Japanese).

Ichikawa, Y. and H. Shikata (1982) Methods for wind field computation from sparse observed wind data, *J. Wind Eng. 14*, 43–52 (in Japanese).

Kitada, T., A. Kaki and M. Kurokawa (1981) Estimation of three-dimensional wind velocity field from observed data, *Preprint, 35th Annual Conf. of JSCE, Part II,* pp. 484–484 (in Japanese).

Nishioka, M., M. Asai and S. Iida (1981) Wall phenomena in the final stage of transition to turbulence, in: *Transition and Turbulence* (Academic Press, New York) pp. 113–126.

Sasaki, Y. (1958) An objective analysis based on the variational method, *J. Meteor. Soc. Japan 36*, 77–88.

Sherman, C.A. (1978) A mass-consistent model for wind fields over complex terrain, *J. Appl. Meteor. 17,* 312–319.