Prediction of flow field in wind farm using a new multiple wake model

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ABSTRACT: This paper aims at developing a new multiple wake model to accurately predict both mean and turbulent flow field in the wind farm. Firstly, numerical simulations are performed for two wind turbines and a small wind farm with six turbines. The characteristics of multiple wakes are systematically investigated. A new multiple wake model is then proposed, in which the local effective turbulence on the rotor and the wake mixing effects are considered. Finally, comparisons of predicted results with numerical simulation data show that the proposed model can favorably predict the mean velocity and turbulence intensity distributions in multiple wake regions.

KEYWORDS: wind farm, wind turbine wakes; wake model; multiple wake superposition.

1 INTRODUCTION

In the wind farm, wakes from multiple turbines lead to a significant wake-turbine interaction as well as the wake-wake interaction. In comparison to single turbine wake, the analytical studies for the multi-wake has not received much attention due to its physical complexity. The commonly used approach in wind farm design is to combine the single wake calculation based on the superposition approach for mean flow field, however, the turbulence was not considered in the multiple wake modelling [1,2]. In the IEC61400-1 for wind turbine design [3], the added turbulence from neighboring turbines are combined for fatigue prediction, while turbulence distribution is assumed constant in the wake and the accuracy of turbulence superposition has not been evaluated. Note that the variation of turbulence has a significant impact on the wake recovery and interactions. This paper aims at proposing a new multiple wake model to improve the prediction accuracy of both the mean velocity and turbulence intensity in the wind farm.

2 NUMERICAL SIMULATION

In this study, the Reynolds Averaged Navier-Stokes (RANS) equations are used to simulate the turbulent wake flows as follows:

\[
\frac{\partial (\rho U_i)}{\partial x_i} = 0
\]

\[
\frac{\partial (\rho U_i U_j)}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \right] + \frac{\partial (\rho U_i U_j)}{\partial x_j} + f_{\bar{u}_i}
\]

where \(U_i\) and \(\bar{p}\) are the mean wind velocity and the pressure respectively. \(\rho\) is density of the fluid and \(\mu\) is the molecular viscosity, \(f_{\bar{u}_i}\) is the rotor force per unit grid volume parameterized by an actuator disk model with rotation (ADM-R). Reynolds Stress Model (RSM) [4] is utilized to express the Reynolds stress tensor \(-\rho u'_i u'_j\) to close the momentum equation.
Two wind turbines under wind directions of 0° and 10° are firstly chosen as two representative layouts to simulate the multiple wakes: full overlap and partial overlap. Finally, a supposed small wind farm with six turbines are simulated to further verify the proposed wake model.

3 A NEW MULTIPLE WAKES MODEL

3.1 Rotor effective onset wind speed and turbulence intensity

Downstream turbines in the wind farm experience a non-uniform inflow over the rotor area, hence in order to apply the wake models, an effective onset wind speed $U_{h,i}$ and local turbulence intensity $I_{a,i}$ on the rotor is evaluated as follows:

$$U_{h,i} = \frac{1}{A_{rotor}} \int_{rotor} U(x_i, y, z) \, dA, \quad I_{a,i} = \frac{1}{A_{rotor}} \int_{rotor} \sigma_u^2(x_i, y, z) \, dA$$

where $A$ is the area of the rotor, $U$ and $\sigma_u$ is the wind speed and turbulence in the wake region.

3.2 Mean velocity

To predict the mean flow field $U$ in the multiple wake region, individual velocity deficits are combined based on the principle of Linear Superposition (LS) [2]:

$$U = U_0 - \sum_{i=1}^{n} \Delta U_i, \quad \Delta U_i = F(C_{T,i}, I_{a,i}, x/D) \phi(r_i / \sigma_i)$$

where $U_0$ is the free stream wind speed, $\Delta U_i$ is the velocity deficit induced by each wind turbine calculated by the single wake model of Ishihara & Qian [5], $F$ and $\phi$ is the streamwise function and spanwise function, respectively, $C_{T,i}$ is the thrust coefficient of turbine $i$, $\sigma_i$ is the representative wake width, $r_i$ is the spanwise distance from the wake center, $D$ is the rotor diameter.

3.3 Turbulence intensity

The individual added turbulence are then superimposed by the principle of Linear Superposition of Square (LSS) [3]:

$$\sigma_u^2 = \sigma_{u,0}^2 + \sum_{i=1}^{n} (\Delta \sigma_{u,i} + \Delta \sigma_{u,ij})^2, \quad \Delta \sigma_{u,i} = G(C_{T,i}, I_{a,i}, x/D) \phi(r_i / \sigma_i)$$

where $\sigma_{u,0}$ is the free stream turbulence standard deviation, $\Delta \sigma_{u,i}$ is the added turbulence from each wind turbine based on the analytical Gaussian wake model [5], $G$ and $\phi$ is the streamwise function and spanwise function, respectively. Note that $\Delta \sigma_{u,ij}$ is a newly proposed correction term for turbine $i$ to consider the wake mixing with the closest upstream turbine $j$ as follows:

$$\Delta \sigma_{u,ij} = \begin{cases} \frac{1}{2} \Delta \sigma_{u,tip} \cos^2 \left( \frac{\pi r_i}{D} \right), & \left( y, z \right) \in A_1, \left| y_j - y_i \right| \leq \frac{D}{4}, \\ -\frac{1}{2} \Delta \sigma_{u,tip} \sin^2 \left( \frac{\pi (y_j-y_i)}{D} \right) \cos^2 \left( \frac{\pi (z-H)}{D_{ij}} \right), & \left( y, z \right) \in A_2, \frac{D}{4} < \left| y_j - y_i \right| \leq \frac{5D}{4} \\ 0, & \text{else} \end{cases}$$

where $\Delta \sigma_{u,tip}$ is the added turbulence at the tip side, $A_1$ and $A_2$ describe the turbulence correction areas for full and partial overlap as follows:

Full overlap: $A_1 = \left\{ (y, z) \mid r_i \leq \frac{D}{2} \right\}$
Partial overlap: $A_2 = \left\{ (y, z) \mid |y_j - y_i| < D, |z - H| \leq \frac{D}{2}, \text{sgn}(y_j - y_i) \text{sgn}(y_j - y_i) = 1 \right\}$
4 RESULTS AND DISCUSSION

Figure 1 shows the horizontal contours and profiles of mean velocity at hub height for two wind turbines, and Figure 2 are the results of six wind turbines. It can be seen that for the full overlap wakes, both velocity deficits and turbulence become stronger, while in the partial overlap condition, the turbulence standard deviation in the overlap areas is weakened due to the wake mixing.

The quantitative comparison of velocity profiles show that the new proposed model based on the principle of LS with effective rotor onset turbulence presents favorable agreement with CFD results, while the conventional multi-zone model with Root Sum Square (RSS) superposition principle [1] generally underestimates deficits in the overlap region and overestimated them in the non-overlap areas. Note that in previous research [2], LS was implemented without considering effective rotor onset turbulence, hence the wake deficit was overestimated.

In addition, from the comparison of turbulence intensity profiles at selected positions, it can be found that the LSS superposition approach with the proposed turbulence correction for wake mixing well predicted the turbulence distribution in the wake region both for two turbines and six turbines. However, the IEC model [3] generally gives conservative prediction in the near wake region for the full overlap wakes and large overestimation in the partial overlap wake regions.

Figure 1. Contours and profiles of mean velocity and turbulence intensity in the wake of two wind turbines: (a), (c), (e) for the wind direction of 0°; (b), (d), (f), (h) for the wind direction of 10°.
5 CONCLUSIONS

(1) The wake characteristics including the mean velocity and turbulence intensity of multiple wind turbines are systematically investigated by numerical simulations.

(2) A new multiple wake model accounting for the local effective inflow on rotor is proposed, in which velocity deficits are combined by Linear Superposition, and turbulences are added using Linear Superposition of Square with a newly proposed correction term to consider the wake mixing. The proposed model is finally verified by comparison with numerical simulation results.

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7 REFERENCES