HYDRODYNAMIC RESPONSE OF A SEMI-SUBMERSIBLE FLOATING OFFSHORE WIND TURBINE: NUMERICAL MODELLING AND VALIDATION

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A fully coupled nonlinear simulation tool using an augmented Morison’s equation is developed to predict the dynamic response of floating offshore wind turbine (FOWT) system. Water tank tests are conducted to investigate the unsteady characteristics of hydrodynamic coefficients and to investigate the performance of the simulation tool in the prediction of dynamic response of FOWT under different sea states. Three important issues regarding to unsteady characteristics of hydrodynamic coefficients, Morison’s equation and dynamic model of mooring system are discussed in this paper. The dynamic responses of FOWT with improved hydrodynamic models agree well with those measured in water tank tests.

**Keywords:** Floating offshore wind turbine system, Dynamic response, Morison’s equation, Unsteady characteristics, Axial Froude-Krylov force, Mooring fairlead tension

**INTRODUCTION**

Potential flow theory and Morison’s equation are widely used to evaluate hydrodynamic loads. Potential flow theory accounts for the Froude-Krylov forces and diffraction effects for a large rigid body. FAST (Fatigue, Aerodynamics, Structures, and Turbulence) developed by National Renewable Energy Laboratory is one of the programs using potential theory to predict the dynamic response of FOWT. Nonlinear drag forces are added to the hydrodynamic force after FAST released its version 8. In contrast to rigid body dynamic response analysis, elastic body dynamic response is performed by modeling the structural components with finite element method (FEM), and the distributed hydrodynamic force acting on each member is evaluated according to Morison’s equation. CAsT (Computer Aided Aerodynamic and Aerelastic Technology), developed by Phuc and Ishihara[1], is one of the simulation tools using Morison’s equation and elastic body model. The commercial program Bladed provides an option to use the potential flow theory or Morison’s equation.

Many studies have been conducted to simulate dynamic response of floating wind turbines. Potential flow theory-based hydrodynamic model can be used to predict the motion response of the platform to a sufficiently accurate level [2, 3]. However, rigid body dynamic response makes it impossible to capture the elastic motion of the structural elements. The study by Phuc and Ishihara [1] highlighted that the dynamic response of flexible floating platform might be underestimated by the rigid body assumption. Therefore, elastic body response analysis is preferred by using Morison’s equation to evaluate the distributed hydrodynamic loads on each structural element. With a full Morison’s equation, Phuc and Ishihara [1], and Waris and Ishihara [4] used a modified Morison’s equation to analyze the dynamic response of a semi-submersible FOWT and showed some differences between predicted response and those from water tank test. This indicates that the conventional Morison’s equation needs to be improved to provide much more accurate prediction of dynamic response of FOWT under wide sea states.

The mooring system is critical for station-keeping of FOWT in sea states. A comprehensive literature review in terms of dynamic modeling and quasi-static modeling of mooring system is provided in the study by Hall et al.[3]. The quasi-static model in the form of either force-displacement relationships or analytical solutions for catenary cables in static equilibrium is used in some simulation tools, such as FAST and Bladed, because of its computational efficiency. However, the effects of dynamic behavior of mooring system on the dynamic motion of the platform and mooring line fairlead tension might be significant in cases where the motion of the platform is significant. Since quasi-static model has shown its deficiency in fairlead tension prediction not only in the model test[3], but also in the field measurement [5], dynamic mooring line models including added mass, mooring line inertia, and nonlinear hydrodynamic drag force should be employed and validated.

Several studies using the dynamic modeling of mooring system have been performed recently[3, 4]. The dynamic analysis for platform-mooring coupling system can be classified into de-coupling and full-coupling analysis. In the study by Hall et al.[3], fairlead tension predicted by de-coupling dynamic model analysis matched well with measured fairlead tension. Since poor fairlead tension agreement was recorded when full-coupling analysis was conducted [3], the accuracy of dynamic modeling mooring system in prediction of fairlead tension using full-coupling analysis should be validated against the water tank test.

**WATER TANK TESTS**

Two water tank tests are carried out in this study to examine the hydrodynamic coefficients of FOWT and to investigate the dynamic response of scaled FOWT under various sea states.

**Forced oscillation tests**

A 1/50 scaled platform used in Fukushima FORWARD
project [6], is forced oscillate in horizontal and vertical directions sinusoidally as in Eq.(1).
\[ x(t) = a \sin(\omega t) \]  
where, \( x(t) \) is the time-varying displacement, \( a \) is the oscillating amplitude.

The time series of hydrodynamic force, \( F_r(t) \), associated with the added mass coefficient and drag coefficient is obtained by subtracting the body inertia force, \( F_i(t) \); hydrostatic force, \( F_h(t) \); and radiation damping force, \( F_d(t) \), from the measured force, \( F(t) \).
\[ F_n(t) = F(t) - F_i(t) - F_h(t) - F_d(t) \]  
As introduced in reference, Fourier averages of \( C_a \) and \( C_d \) are then obtained as follows:
\[ C_a = \frac{1}{\pi a \omega A} \int_0^T F_r(t) \sin(\omega t) dt \]  
\[ C_d = -\frac{3}{4 \rho A a^2} \int_0^T F_r(t) \cos(\omega t) dt \]

In this study, one towing test is also carried out to evaluate the drag coefficient in a steady flow. The drag coefficient is obtained as follows:
\[ C_d = \frac{F_d}{0.5 \rho U^2 A} \]
where, \( F_d \) is the measured drag force on the platform in towing direction, \( U \) refers to the towing speeds (0.2m/s, 0.5m/s and 1.0m/s), and \( A \) is the characteristic area in the horizontal direction.

The drag coefficient obtained from three different towing speeds are then averaged as shown in Fig.1(a), which corresponds to infinite period.

In the numerical simulation of the dynamic response of FOWT to regular waves or irregular wave, the hydrodynamic coefficients at corresponding wave period or peak period are employed.

![Hydrodynamic coefficients](image1)

(a) In horizontal direction  
(b) In vertical direction

Fig.1. Hydrodynamic coefficients in (a) horizontal direction and (b) vertical direction obtained from forced oscillation tests.

![Hydrodynamic force on semi-submersible model](image2)

Fig. 2. The 1:50 scaled semi-submersible model used in the water tank test.

Dynamic response tests

Fig. 2 shows the model used in the dynamic response test. The translational motion of the platform in X, Y, and Z directions are called surge, sway, and heave motion, while the rotational motion about X, Y, and Z axes are named roll, pitch, and yaw motion respectively.

NUMERICAL MODELS

Equation of motion

The general nonlinear time domain equations of motion for the coupled wind turbine and support platform system is written as follows:
\[ \mathbf{M}(\mathbf{x}) + \mathbf{C}(\mathbf{x}) + \mathbf{K}(\mathbf{x}) = \mathbf{F}_0 + \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 \]  
where, \( \mathbf{M} \), \( \mathbf{C} \), and \( \mathbf{K} \) are the mass matrix, damping matrix and stiffness matrix of the system respectively; \( \mathbf{x} \), \( \dot{\mathbf{x}} \), and \( \ddot{\mathbf{x}} \) are the unknown displacements in six DOFs and their time derivatives; \( \mathbf{F}_0 \) is the gravitational force; \( \mathbf{F}_1 \) refers to the buoyancy force; \( \mathbf{F}_2 \) means the hydrodynamic force; \( \mathbf{F}_3 \) stands for the force from mooring line system; and \( \mathbf{F}_4 \) represents the restoring force.

Hydrodynamic force

The Morison’s equation is used to evaluate the hydrodynamic force in both normal and tangential direction. Illustration of force acting on a segment of slender cylinder and heave plate is shown in Fig. 3.

![Hydrodynamic force on cylinder and heave plate](image3)

Fig. 3. Illustration of hydrodynamic force acting on segment of cylinder and heave plate.

Hydrodynamic force acting on the element in normal direction is expressed by the relative form of modified Morison’s equation as follows:
\[ F_i = \rho \omega^2 t u' + C_d \rho \omega^2 u' + 0.5 \rho \omega^2 C_p A f(\vec{u}' \cdot \vec{x}) |\vec{u}' \cdot \vec{x}| - C_r \rho \omega^2 \vec{x} \cdot \mathbf{C} \vec{x} \]  

A correction factor is obtained by comparing the drag coefficient in a steady flow with measured one in the forced oscillation test. The correction factor for the drag coefficient in the normal direction is identified as follows:
\[ r_d = \frac{C_{a}}{\sum(C_{a}\sin^2(\theta_i))} \]
where, \( C_a \) is the measured drag coefficient in the horizontal direction shown in Fig.1(a), and \( \theta_i \) is the angle between the axial and global X directions of the element.

The correction factor for the drag coefficient of heave
plate in its tangential direction is obtained as follows:

$$r'_r = \frac{C_r \cdot \sum (C_r^x A_r^x) - \sum (C_r^x A_r^x \sin(\theta))}{\sum (C_r A_r)} \quad (9)$$

The correction factor for the added mass coefficient in the normal direction is evaluated as follows:

$$r'_n = \frac{C_n^y \cdot \sum (C_n^y A_n^y) - \sum (C_n^y A_n^y \sin(\theta))}{\sum (C_n A_n)} \quad (10)$$

The correction factor for the added mass coefficient of heave plate in its tangential direction is identified as follows:

$$r'_h = \frac{C_h^y \cdot \sum (C_h^y A_h^y) - \sum (C_h^y A_h^y \sin(\theta))}{\sum (C_h A_h)} \quad (11)$$

**NUMERICAL RESULTS AND DISCUSSION**

**Effect of unsteady characteristics of hydrodynamic coefficients**

Fig. 4 (a) shows the measured and predicted time series of sway motion in the free decay test. It is found that both the amplitude of sway motion and natural period in sway mode are overestimated when the hydrodynamic coefficients determined from the steady flow are not corrected, while both the amplitude and natural period are improved when the hydrodynamic coefficients are corrected according to the results obtained from forced oscillation test.

![Figure 4](image)

(a) Sway motion  (b) Natural period

Fig. 4. Comparison of measured and predicted time series of (a) sway motion and (b) natural periods of the FOWT obtained from the free decay tests.

Natural periods of floating system obtained from free decay tests are shown in Fig. 4(b). The predicted natural periods in six DOFs match well with those obtained from the water tank test. The predicted natural period difference between quasi-static and dynamic models is negligible because the added mass provided by mooring system is sufficiently small compared with the mass of the total floating system.

**Effect of axial Froude-Krylov forces**

The motion RAO in the regular waves is expressed as follows:

$$\frac{A_{O}^n}{A_{O}^n} = \frac{A_{O}^n}{A_{O}^n}; \quad \frac{A_{O}^n}{A_{O}^n} = \frac{A_{O}^n}{A_{O}^n}; \quad \frac{A_{O}^n}{A_{O}^n} = \frac{A_{O}^n}{A_{O}^n} \quad (12)$$

Fig. 5 shows the measured and predicted dynamic RAOs and phase difference between the motion of platform and the incident wave. It is found that the surge RAO increases linearly with increasing wave period since the conducted wave period is far away from the surge natural period of floating system. In addition, dynamic motions are improved in the whole wave range when the axial F-K forces on slender members are considered, especially in the heave and pitch directions as shown in Fig. 5(b) and (c). Phase differences shown in Fig. 5 between the incident wave and dynamic motion are improved as well. An accurate prediction of phase difference is of great importance in the evaluation of dynamic motions of FOWT since the hydrodynamic force is not only dependent on the incident wave but is also associated with the motion of platform itself.

![Figure 5](image)

(a) Surge RAO  (d) Phase difference in surge

(b) Heave RAO  (e) Phase difference in heave

(c) Pitch RAO  (f) Phase difference in pitch

Fig. 5. Measured and predicted dynamic motion RAO and phase differences between the motions of platform and incident wave in regular waves.

**Effect of dynamic behavior of mooring system**

Fig. 6 shows the measured and predicted fairlead tension RAO and phase difference in the mooring lines expressed by ML1 and ML3. It is found that the quasi-static model apparently overestimates the fairlead tension RAO in all wave periods for ML1 and ML3, while the dynamic model provides a better prediction. In addition, accuracy of predicted phase differences by means of the dynamic model is improved as well. This is primarily due to the consideration of nonlinear drag force in the dynamic model. The fairlead tension in ML3 is
overestimated by dynamic model, which is due to the effect of the attached cable in the experiment, and it is improved by considering this effect.

![Fig. 6. Measured and predicted fairlead tension RAO and phase difference in regular waves.](image)

The measured and predicted time series and PSD of fairlead tension in mooring line 1 by the quasi-static and dynamic models are shown in Fig. 7 for the case with a wave period of 2.4s. As observed in Fig. 7(a), the quasi-static model overestimates the magnitude of fairlead tension in mooring line 1 by 51% and phase lag is observed between the measured and predicted fairlead tension. In addition, harmonic components are observed in the measured fairlead tension. From the PSD shown in Fig. 7(b), harmonic components are observed from the measurement across the whole frequency domain. Among these peaks, the first one (0.09 Hz) corresponds to the natural frequency of the surge motion of the platform and the second one (1F = 0.42 Hz) is excited by the motion owing to the first-order hydrodynamic loads on the platform and by the resonance of the heave motion of the platform (0.42 Hz). The third and fourth peaks are associated with the motion component caused by the nonlinear drag force acting on the platform and mooring lines, respectively. The quasi-static model could predict the components of the first three peaks, but, it fails to yield the higher harmonic components, such as 3F as shown in Fig. 7(b). However, the dynamic model successfully predicts all harmonic components. In addition, the peaks predicted by the dynamic model match well with those observed from the measurement, while the first three peaks by the quasi-static model are overestimated. As a result, the influence of the dynamic behavior of the mooring line is significant in the evaluation of fairlead tension amplitude.

**CONCLUSIONS**

A fully coupled nonlinear simulation tool, CAaST, is developed with augmented Morison’s equation, and the predicted dynamic responses of FOWT are validated by two water tank tests. The conclusions obtained are as follows:

1. Correction factors for added mass and drag coefficient are proposed and used in the augmented Morison’s equation to account for the unsteady characteristics of hydrodynamic coefficients.

2. Axial Froude-Krylov loads on slender members are crucial in prediction of dynamic response of FOWT. It makes a difference in dynamic responses for the heave and pitch motions even though the axial Froude-Krylov force takes only a small fraction of total force.

3. The dynamic model successfully reproduced all the harmonic components of fairlead tension measured by the water tank test, but the quasi-static model only reproduced the first three peaks which are caused by the motions of the platform. Dynamic model shows good agreement with the water tank test. This indicates that inertia and nonlinear drag force on the mooring system have to be considered when evaluating the fairlead tension in mooring lines.

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