ABSTRACT

Floaters with single wind turbine are subjected to large wave induced response, which leads to non-linear behavior of restoring force and mooring system. Therefore, accurate dynamic response prediction considering nonlinear mooring system is needed. In this study, a non-linear finite element model is developed considering dynamic coupling of wind turbine, floater and mooring system. The model employs Morison's equation with Srinivasan's model for hydrodynamic force and a non-hydrostatic model for restoring force.

First, it was observed through a water tank experiment that simple hydrostatic model underestimates the heave response after the resonance peak, while non-hydrostatic model shows good agreement with water tank experiments, verifying the non-hydrostatic model.

Next, the developed model was used to discuss the influence of heave plates on floater response. Heave plates reduced the heave response by shifting the resonance peak to longer period. The reduction of the response increases with increase in heave plate size.

Finally, the applicability of simplified linear modeling of mooring system was investigated using a nonlinear model for catenary and tension legged mooring. The linear model in which the mooring is modeled as a linear spring, was found to provide good agreement with the nonlinear model for tension legged mooring while it overestimated the surge response for catenary mooring.

1. Introduction

The design for Floating Offshore Wind Turbine Systems (FOWTS) is economy driven compared to conventional safety driven offshore structures, requiring them to be much lighter built with slender elements which may induce elastic deformation and large response. This also leads to large aerodynamic, hydrodynamic damping and nonlinear behavior of the mooring system.

There are many existing studies on the dynamic response of FOWTS. Henderson[1] is one of the first to work on floating wind turbine systems. He investigated the contribution of floater motion to wind turbine tower and blade loads. He applied Morison’s equation to large floating systems ignoring hydrodynamic damping and elastic effects. Ishihara and Phuc[2][3] investigated a multi-turbine floater with slender elements to discuss resonance effects due to elastic deformation. They also investigated the contributions of hydrodynamic and aerodynamic damping, discussing importance of these factors through comparison with water tank experiment. These studies[1][2][3], however, used linear model for mooring system and employed linear model for restoring force that can have significant effect on small semi-submersible floater having large response. Jonkman[4] recently discussed dynamic response for a single turbine barge floater system using an analytical model for catenary mooring system, but the accuracy of the model is not validated through experiment. In view of these studies, it is observed that use of nonlinear models for estimation of all applied forces in a coupled simulation is still required, and the effect of different types of mooring system models on dynamic responses also needs to be investigated.

In this study, a nonlinear finite element model is developed to investigate the dynamic response of FOWTS considering full dynamic coupling between wind turbine, floater and mooring system. Section 2 discusses the scheme of the numerical model developed in this study. Section 3 describes the water tank experiment carried out to verify the developed model. Finally the effects of heave plates and nonlinear mooring are discussed in Section 4.
A full finite element model including floater, wind turbine and mooring system was modeled using beam, truss and spring elements.

2.1 Equation of motion

The equation of motion for the floater system can be written in total Lagrangian formulation as

\[ M\ddot{x} + C\dot{x} + Kx = f \]  

(1)

The external force vector \( f \) consists of gravitational force, buoyancy force, hydrodynamic force, restoring force, seabed contact force and aerodynamic force. In addition to these forces, mooring force also needs to be estimated for linear modeling of mooring system.

2.2 Hydrodynamic force model

Morison’s equation [5] is well known for estimation of wave exciting force on slender bottom mounted cylinders. For a floating structure, free to oscillate in waves and current, Morison’s equation is modified by Sarpkaya & Isaacs [6] as follows:

\[ f_{hm} = f_{hm} + f_{hw} + f_{hd} \]  

(2)

\[ f_{hm} = -M_s \ddot{x} \quad \text{where} \quad M_s = \rho \left( C_m - 1 \right) A \]  

(3)

\[ f_{hw} = \rho \sqrt{\frac{C_m}{A}} \]  

(4)

\[ f_{hd} = 0.5 \rho \sqrt{C_m} \left| \dot{u} - \dot{x} \right| \]  

(5)

The force is a sum of added inertia force, Froude-Krylov Force and drag force respectively. Henderson et al. [1] and Offshore Standard DNV-OS-J101 [7] showed that \( C_D \) and \( C_M \) are functions of Keulegan-Carpenter number \( \frac{D_u}{D} \approx \frac{\pi H}{D} \) and relative roughness.

As the Morison’s equation cannot predict the vertical force on base of submerged cylinder, the hydrodynamic force on cylinder base was modeled as sum of added inertia force and linear damping force in this study as follows

\[ f_u = f_{hm} + f_{hd} \]  

(6)

Haslum [8] has proposed that volume of half sphere of water under a vertical cylinder should be considered for added mass as shown in eq. (7), where \( D \) is diameter of the cylinder base.

\[ V = \frac{2\pi}{3} \left( \frac{D}{2} \right)^3 \]  

(7)

In this study, a linear damping ratio of 15% is used following Srinivasan et al. [9]. This damping force can be modeled by a pseudo structural damping of the system as represented in eq. (8).

\[ f_{hd} = -C_{ed} \dot{x} \]  

(8)

2.3 Restoring force model

In order to estimate restoring force due to vertical displacement of the floater, a non-hydrostatic model [NHM] is proposed and its accuracy compared to simple hydrostatic model [HM] is verified through comparison with experiment. The restoring force \( f_r \) in eq. (1) can be estimated using these two models as follows:

\[ f_r |_{HM} = K_R x \]  

(9)

\[ f_r |_{NHM} = K_R (x - \eta) \]  

(10)

\[ K_R = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\rho_g A_v & 0 & 0 \\
0 & 0 & -W \times GM_x & 0 & 0 \\
0 & 0 & 0 & -W \times GM_y & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix} \]  

(11)
2.4 Mooring force model

The linear mooring model used in this study is modeled as external force proportional to the displacement. This method is commonly used in experiment and in simulation due to its ease of modeling.

The nonlinear model for catenary mooring system needs to consider the interaction between mooring line and seabed. The contact force $f_c$ acts on portions of the mooring system in contact with seabed. This force is divided into normal $f_n$ and tangential $f_t$ components with respect to the mooring line and estimated as follows:

$$f_c = \begin{bmatrix} f_t \\ f_n \end{bmatrix} = \begin{bmatrix} 0 & \mu k \\ 0 & k \end{bmatrix} \begin{bmatrix} U \\ V \end{bmatrix}$$  \hspace{1cm} (12)

where $\mu$ is coefficient of friction, $U$ and $V$ are relative displacements in tangential and normal direction respectively and $k$ is penalty constant. According to Ju et al.[9], eq. (12) can be modified to eq. (13) for the condition that $U \approx 0$.

$$f_c = \begin{bmatrix} f_t \\ f_n \end{bmatrix} = k \begin{bmatrix} \mu^2 & \mu \\ \mu & 1 \end{bmatrix} \begin{bmatrix} U \\ V \end{bmatrix}$$  \hspace{1cm} (13)

The contact force at each node lying on seabed is estimated through eq. (13) and Newton-Raphson iterations are used for convergence. The nonlinear model for tension leg mooring needs to consider the effect of element pre-tension, which is done using the model explained in Cook et al.[12].

2.5 Aerodynamic force model

The developed model considers aerodynamic loads acting on wind turbine using quasi-steady theory, blade element and momentum theory, considering blade tip loss, hub loss and tower shadow. Details of these models are described in the references [6][13]. However, this study does not consider wind, as influence of aerodynamic damping has already been discussed in previous studies[2][3]. The wind turbine blades are therefore not required and weight of blades is included in the nacelle.

2.6 Numerical scheme

The equation of motion is solved by direct numerical integration in the time domain using Newmark-$\beta$ method. Newton Raphson method is used for convergence calculation. For structural damping, Caughey Series[14][15] is used.

$$[C] = [M] \sum_{k=0}^{p} \alpha_k \left( [M]^T [K] \right)^k \zeta_i = \frac{1}{2} \frac{a_0}{\omega_0} + a_1 \omega_1 + a_2 \omega_1^2 + \ldots + a_p \omega_1^{p-3}$$  \hspace{1cm} (14)

Where $\omega_0$ and $\zeta_i$ are natural frequency and damping for the $i^{th}$ mode. In this study, a structural damping of 0.50% corresponded to floater and the series is only considered up to 2$^{nd}$ order ($p = 2$).

2.7 Finite element model

Finite element model for the floater system as shown in Figure 1 is prepared. The blades are not modeled as explained before. The mooring arrangement for the experiment (Figure 1) and two types of mooring systems (Figure 2) are also modeled. The Distribution of elements is listed in Table 1. Mass is considered to be concentrated on nodes.

For the wave condition, regular waves based on linear Airy wave theory [6] with 4cm and 12cm wave height are considered. Two wave heights correspond to rated and extreme sea condition respectively. Three heave plate models i.e, model without heave plates and two models with 12cm and 16cm heave plates (Figure 2) are used. The values of $C_D$ and $C_M$ used in simulation are given in Table 2. For the mooring lines, $C_D$ and $C_M$ values are 2.6 and 2.0 respectively[8].
Table 1 Description of FE-model used in the study

<table>
<thead>
<tr>
<th>Component</th>
<th>Description</th>
<th>No. of Element</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind Turbine</td>
<td>Tower</td>
<td>13</td>
<td>Beam</td>
</tr>
<tr>
<td>Floater</td>
<td>Experimental Setup</td>
<td>109</td>
<td>Beam</td>
</tr>
<tr>
<td>Mooring System</td>
<td>Elastic Band</td>
<td>4</td>
<td>Spring</td>
</tr>
<tr>
<td></td>
<td>Kevlar</td>
<td>24</td>
<td>Truss</td>
</tr>
<tr>
<td></td>
<td>Catenary Mooring</td>
<td>30 / line</td>
<td>Truss</td>
</tr>
<tr>
<td></td>
<td>Tension Leg Mooring</td>
<td>10 / tether</td>
<td>Pre-stressed Beam</td>
</tr>
</tbody>
</table>

(a) Plan view  (b) Front view  (c) 3D-view

Figure 1 Floater model

(a) Catenary  (b) Tension Leg  (c) D=8m  (d) D=12m  (e) D=16m

Figure 2 Types and arrangement of Mooring systems and heave plates

Table 2 \( C_D \) and \( C_M \) values used in the simulations

<table>
<thead>
<tr>
<th>Wave Height</th>
<th>D (m)</th>
<th>( K_c=\pi H/D )</th>
<th>( C_D )</th>
<th>( C_M )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Inner Bracing</td>
<td>Outer Bracing</td>
<td>Central Column</td>
</tr>
<tr>
<td>4 m</td>
<td></td>
<td>1.5</td>
<td>2.5</td>
<td>6.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8.40</td>
<td>5.00</td>
<td>2.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.72</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.97</td>
<td>2.00</td>
<td>2.00</td>
</tr>
<tr>
<td>12 m</td>
<td></td>
<td>25.10</td>
<td>15.10</td>
<td>6.30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.71</td>
<td>0.83</td>
<td>0.66</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.72</td>
<td>1.87</td>
<td>2.00</td>
</tr>
</tbody>
</table>

3. Validation of the developed model

Since validating the nonlinear numerical model developed in this study by a single experiment is difficult, three different experiments were carried out. A water tank experiment
discussed in 3.1 is performed to verify the performance of the non-hydrostatic model. Contact model for catenary chain and initial tension model for tension leg mooring are verified in section 3.3.

3.1 Water tank Experiment

Considering Froude’s Number similarity, 1:100 scale is selected for the experiment. The physical properties of the three models are listed in Table 3. The values enclosed in brackets represent values measured during the experiment.

The experiment is carried out in a 1.5 m depth water tank [17]. Figure 3 shows the model layout in the water tank. The model is connected to elastic bands through Kevlar thread which is connected to the tension meters on the front side. On the opposite side, a pulley and weights are used to adjust the required initial tension to 2.95 N that produces a linear stiffness of 45 N/m in surge direction. The linear mooring stiffness used in the experiment is estimated for the catenary mooring arrangement shown in Figure 2(a) using steady wave and current analysis. As the experiment is unidirectional and mooring stiffness in heave direction is negligible, only stiffness in surge direction is considered. Regular waves with period range of 0.6~3.0 sec are considered at intervals of 0.20 sec.

Table 3 Properties of the Experiment Model

<table>
<thead>
<tr>
<th>Heave Plate</th>
<th>Corner Column Base Diameter (cm)</th>
<th>Weight (kg)</th>
<th>Moment of Inertia (Kg-m²)</th>
<th>Center of gravity * (cm)</th>
<th>Center of Buoyancy * (cm)</th>
<th>Meta-centric Height (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>8.0</td>
<td>3.80</td>
<td>0.198</td>
<td>5.13</td>
<td>11.30</td>
<td>8.40 (8.80)</td>
</tr>
<tr>
<td>12 m</td>
<td>12.0</td>
<td>4.06</td>
<td>0.211</td>
<td>5.93</td>
<td>11.70</td>
<td>8.10 (8.30)</td>
</tr>
<tr>
<td>16 m</td>
<td>16.0</td>
<td>4.375</td>
<td>0.230</td>
<td>6.90</td>
<td>12.20</td>
<td>7.70 (8.00)</td>
</tr>
</tbody>
</table>

* With reference to still water level

Measured data

![Model setup](image)

Figure 3 Water tank experiment setup

3.2 Non-Hydrostatic model for restoring force

Prior to the verification of the non-hydrostatic model, it is essential to verify the FE-model used in the simulation. Free vibration tests are carried out to estimate the natural periods of floating wind turbine system in surge and heave modes. Similarly, free vibration test are simulated using the FE-model. Estimation of \( C_D \) and \( C_M \) values for FE-models in free vibration test is based on element velocity ‘U’ and period ‘T’ estimated from the free vibration experiment. Figure 4 shows comparison of observed and predicted natural period in surge and heave modes for prototype scale. The error bars represent variation in experimental results over a set of 20 observations. The maximum difference in predicted and observed is about 5%. The FE-models are therefore, in acceptable agreement with the actual experiment models.
To verify the performance of non-hydrostatic restoring force model, it is compared with the hydrostatic model and dynamic pressure model. The comparison is performed considering the original floater (without heave plates).

Figure 5 shows the comparison of surge, heave and pitch response amplitudes for the three models with experiment data. Surge responses for the three models provide similar results, showing that the restoring force model has no effect on the surge response. Heave response after resonance peak is underestimated by the hydrostatic model while it is overestimated by dynamic vertical pressure model. The non-hydrostatic model gives better agreement with experiment compared to the other models for heave. Pitch response is normalized with respect to floater span ‘B’ and wave height ‘H’. It can be observed that dynamic pressure model and non-hydrostatic model provide similar results while hydrostatic model overestimates pitch near resonance state.

The corresponding mooring tension amplitudes are also presented in Figure 6. The tension amplitude is normalized by the product of hydrodynamic pressure ‘$p_0$’ at the water surface and the projected area of the floater ‘$BS$’, where ‘$B$’ refers to floater span and ‘$S$’ to submerged depth. The dynamic wave pressure ‘$p_0$’ at water surface is estimated using linear wave theory as ‘$p_0=\rho g H/2$’. The three models provide similar agreement with experiment for mooring tension since line tension is independent of vertical response in the experimental arrangement considered.

The non-hydrostatic model is therefore clearly the better choice for response prediction of such small sized floaters.
3.3 Nonlinear model for mooring system

The performance of the contact model is verified through an experiment using a catenary chain. A chain 11.6 g/m in weight and 2.0 m in length is suspended with horizontal span of 1.5 m as shown in Figure 7(a). A plate is raised from underneath to achieve different contact lengths between the plate and chain. Four plate elevations of 0.55, 0.50, 0.45 and 0.403 m are considered, where elevation is measured downwards from support. Developed model is used to reproduce this catenary arrangement using 80 truss elements. Figure 7(b) shows the comparison of profile from experiment and simulation for free catenary and for plate elevation $Z = 0.403$ m, while Figure 7(c) shows the comparison of contact lengths at four plate elevations. The comparison indicates that the developed tool can correctly model the chain profile and contact lengths for the catenary chain.

For nonlinear modeling of tethers in Tension Leg mooring system, tether pretension is very important. The developed tool can consider element pretension and its performance is verified using experimental data from Kanda et al. [18]. The reference experiment was performed on a 1:100 scaled submerged tether model having length of 4.391 m and initial pre-tension of 21.60 N. An FEM of the tether is prepared using 20 pre-tensioned beam elements. The tether is subjected to a harmonic forced vibration of 100 mm amplitude at 1.28 sec, which is the first mode of vibration. The comparison of tether profiles over a cycle of vibration at interval of 0.133 sec is shown in Figure 8. It can be observed that the developed model shows good agreement with the experiment.
4. Effects of heave plates and nonlinear mooring

4.1 Effect of heave plates on floater dynamic response

Figure 9 shows the comparison of surge, heave and pitch response for the three heave plate models. The surge and pitch response for the three models show similar trends, indicating that heave plates have small effect on these modes.

The heave response demonstrates a clear shift in resonance peak with inclusion of heave plates, resulting in the reduction of the response around resonance. This is due to the increase in added mass which is a function of heave plate diameter. Therefore it is possible to reduce the heave motion around resonance by installing heave plates.

4.2 Influence of mooring systems on floater dynamic response

Now, influence of mooring system model is discussed using linear and nonlinear model for catenary and tension legged mooring systems. The specification of the simulation conditions are described in Tables 4 and 5.

<table>
<thead>
<tr>
<th>Table 4. Models used in the simulation for mooring model comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
</tr>
<tr>
<td>Catenary</td>
</tr>
<tr>
<td>Tension legged</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 5. Specification of the models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wave height</td>
</tr>
<tr>
<td>Sea depth</td>
</tr>
<tr>
<td>Catenary chain</td>
</tr>
<tr>
<td>Tension leg</td>
</tr>
<tr>
<td>Heave plate</td>
</tr>
</tbody>
</table>
4.2.1 Effect of nonlinear catenary mooring model

Figure 10 shows the comparison of response for the catenary mooring model and the tension legged mooring model under 4 m regular wave. Figure 10 (a)-(c) show the predicted surge, heave, and pitch response amplitude of the floater for the catenary mooring model. The surge response using linear model is flattened out in the nonlinear model. This is because of the nonlinear behavior of the mooring system. The mooring stiffness increases with increase in floater’s surge amplitude and changes the natural period of the system, hence eliminating the resonance effect observed using the linear model. The heave response is quite similar for both models as catenary mooring system does not strongly restrain the vertical motion. The slightly reduced heave response amplitude at resonance peak for the nonlinear model is caused by drag and inertia effect on mooring line affecting the heave response. The pitch response is also slightly overestimated by the linear model and is observed to resonate with surge response and has overestimation, as can be expected.

Therefore, a single linear catenary mooring model cannot be directly used over all wave period range since it cannot consider the dynamic component in the mooring tension, resulting in these underestimations.

4.2.2 Effect of nonlinear tension legged mooring model

The linear model used here considers initial tension and axial stiffness for estimation of vertical linear mooring stiffness using \( T_e / L + E A / L \), where ‘L’ represents the length of the tether. The dynamic component of tension is estimated using this linear stiffness and floater displacement. Horizontal mooring stiffness can be expressed as \( T_e / L \). The mooring arrangement and sectional properties of tether are based on Shimada et al. [19]. The mooring arrangement consists of two tethers connected to each corner floater, each having an initial tension \( T_e = 1145 \text{ KN} \) and \( A = 9270 \text{ mm}^2 \).

Figure 10 (d)-(f) show the predicted response for the tension legged mooring model. The heave and pitch response are much smaller than the surge due to strong restraint. Here it is observed that the linear model provides very good agreement with the nonlinear model. This indicates that the contribution of the dynamic tension to mooring stiffness is negligible.

5. Conclusions

A fully coupled nonlinear FEM model is developed to predict the dynamic response of FOWTS with non-hydrostatic model and nonlinear mooring model. The conclusions are summarized below.
• The model developed in this study was verified through comparison with water tank
tests and showed good agreement. The hydrostatic model underestimates heave
response after resonance peak due to the underestimation of restoring force.
• Heave plates reduce the heave response by shifting the natural period of the system
to longer period. This effect increases with increase in heave plate size.
• The comparison of dynamic response between linear model and nonlinear model for
catenary and tension legged mooring system shows that simplification of mooring
stiffness to linear spring causes overestimation of surge response at resonance
state for catenary mooring system but provides good agreement for tension legged
mooring system. This is because of the large nonlinearity in catenary mooring due to
large dynamic component in mooring tension, and small nonlinearity for tension
legged mooring due to large initial tension.

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