

# A Numerical and Semi-theoretical Formulation of Seismic Response Analysis of Wind Turbines in Time Domain

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## Objective

The prediction of seismic response of wind turbines becomes of great importance when wind farms are designed and developed in seismically active regions. Being a seismically active region, Japan has strict design procedure to avoid collapse under seismic excitation. Stability of wind turbine structures against level II earthquakes is required, which is carried out depending upon tower height either by time domain analysis or by design formula.

Unlike wind loads, a wide range of frequencies are involved in seismic waves that may excite higher modes of the wind turbine system. Therefore, it becomes important to include higher modes for predicting the response characteristics of wind turbines under seismic loads. To simulate the response of wind turbines, modal method is widely used in the field of wind energy engineering. In modal method, displacements are expressed as linear combination of mode shapes and the equations of motion are simplified as a set of single degree of freedom equations. Bosanyi (Bladed, 2005) used the modal method and the whole wind turbine is divided into two substructures: one is the rotor, and other is the tower. Since rotor and tower are calculated separately, it requires coupling between them to model interaction between rotor and tower. However, this coupling is not sufficient in the modal model because only very limited degrees of freedom are modeled.

In this study, a full nonlinear FEM model is developed that takes into account the geometric nonlinearity and coupling between the rotor and tower. Aim of present study is to examine response characteristics of wind turbines in time domain, and to formulate equations for prediction of seismic loads using response spectrum. First response spectrum according to Japanese building standard are introduced and process of earthquake wave generation is discussed. Then investigations on modal contribution to response of wind turbine are discussed, and finally semi theoretical formulation for seismic load estimation is discussed.

## Seismic Conditions

- Japanese design code requires evaluation of wind turbine structural stability against level II earthquakes that correspond to 500 - 1000 years return period.
- Earthquake waves generated according to specification are usually used to conduct stability analysis of tall structures. Phase of real seismic waves is used to generate seismic waves for the desired type of response spectrum.
- In this study, three representatives of earthquakes are chosen for the phase component i) standard waves such as El Centro and Taft waves, ii) in-land and off shore waves such as Kobe and Hachinohe waves, and iii) Local wave such as Tohoku wave.

For level II earthquake, the response spectrum at the engineering bed rock is defined (SBL 2004). A soil amplification factor and damping correction factor are used to obtain the response spectrum at ground surface and of structure with specified damping ratio.

Spectrum at engineering bed rock,

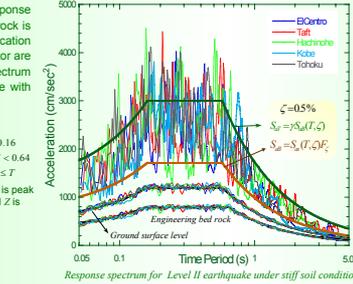
$$S_{a0}(T, 0.05) = \begin{cases} a_0(1+9.375T)Z & T < 0.16 \\ 2.5a_0Z & 0.16 \leq T < 0.64 \\ (1.6a_0/T)Z & 0.64 \leq T \end{cases}$$

where Z is regional factor,  $a_0$  (0.32m/s<sup>2</sup>) is peak acceleration at engineering bed rock and Z is regional factor.

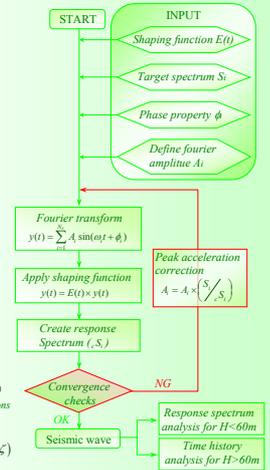
Acceleration response spectrum at ground surface,

$$S_{a0}(T, \zeta) = S_{a0}(T, \zeta)G_r$$

$$G_r = \begin{cases} 1.5 & T < 0.576 \\ 0.864 & 0.576 \leq T < 0.64 \\ 1.35 & 0.64 \leq T \end{cases}$$



SDOF response spectrum,  
Building response spectrum:  $S_{a0}(T, \zeta) = S_{a0}(T, \zeta)F_c$  and  $F_c = 1.5(1+10\zeta)$   
Tower response spectrum:  $S_{aT}(T, \zeta) = \gamma S_{a0}(T, \zeta)$



## Numerical Modeling

- The equation of motion of the FOWTS is written as follows.

$$[M]\ddot{X} + [C]\dot{X} + [K]X = [F_a] + [F_s]$$

Where [M], [C], [K] is a mass, damping, stiffness matrix; [F<sub>a</sub>] and [F<sub>s</sub>] are the aerodynamic force and the seismic force, respectively, and X is unknown vectors containing translations.

Dynamic analysis	The Newmark-Beta method
Eigenvalue analysis	Subspace iteration procedure
Element type	Beam element
Formulation	Total Lagrangian formulation
Geometric non linearity	Newton-Raphson method
Aerodynamic force	BEM Theory
Damping	Rayleigh damping

Numerical scheme

- Two wind turbine systems, 400kW and 2MW, were modeled using beam elements with 58 nodes and 57 elements for full FEM model. For WEE, same number of tower and nacelle elements are used with rotor and nacelle masses lumped at the respective nodes.

Tower Modes	2MW Turbine		400kW Turbine		% e
	FEM	WEE	FEM	WEE	
1 <sup>st</sup>	2.584	2.534	1.93	1.232	0.57
2 <sup>nd</sup>	0.347	0.347	0.06	0.151	1.33
3 <sup>rd</sup>	0.128	0.125	2.34	0.056	0.057

Natural frequencies of modal wind turbines

## Seismic Response Analysis in Time Domain

### Contribution of higher modes

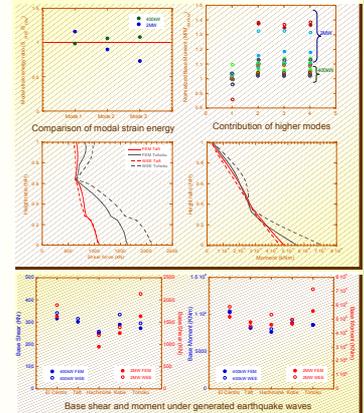
It is necessary to understand the effect of modeling methods, e.g., FEM and WEE, on the response characteristics of system. Eigen vector analysis and dynamic response analysis are conducted to investigate the contribution of higher modes towards the response of wind turbine models.

- The first mode shapes agree well for both models. However, second modes and third modes have shown different behavior for the two models.
- Modal strain energy ratio, calculated from eigen vectors, suggests that both WEE and FEM would result in similar response for the 400kW turbine. However larger contribution of higher modes is observed in case of 2MW turbine.
- Base moment for the two models agrees well for the 400kW turbine and show smaller contribution of higher modes. In case of 2MW turbine, high contributions from 2<sup>nd</sup> mode occurs for Taft and Kobe waves that have significantly excited the second mode of wind turbine.
- The consistency of the seismic load profile, obtained by FEM and WEE model, is significantly influenced by the type of earthquake phase used.

### Dynamic Response Characteristics

To investigate the response behavior of wind turbine systems, a dynamic response analysis was carried out for waves generated from the target spectrum of Level II.

- WEE model tends to overestimate the base reactions for dynamic analysis, which becomes as large as 28% in case of 2MW turbine.
- A close agreement between the simulated seismic loads exists for two models of 400kW, that is consistent with outcome of the modal strain energy analysis.



## Seismic Load Analysis by Response Spectrum

### Estimation of vertical load profile

A linear distribution of the shear force and moment profile is assumed. Contribution of first mode are calculated using response spectrum and those of higher modes is formulated based on the FEM analysis. The profile of mean loads acting on the tower can be obtained using following expressions (JSCE 2007):

$$Q_z = 0.64(1 + Cs)S_a(T, \zeta) \sum_{i=1}^n m_i A_i$$

$$M_z = 0.64(1 + Cs)S_a(T, \zeta) \sum_{i=1}^n m_i A_i h_i$$

where

$$\text{Distribution factors: } A_i = 1 - 0.2(z_i/H_i); \quad A_n = 1 - 0.97(z_n/H_n)$$

$$\text{Shear modification factor: } C_s = \begin{cases} 0 & (T \leq 0.7s) \\ 0.075(1 - e^{-0.7s}) & (T > 0.7s) \end{cases}$$

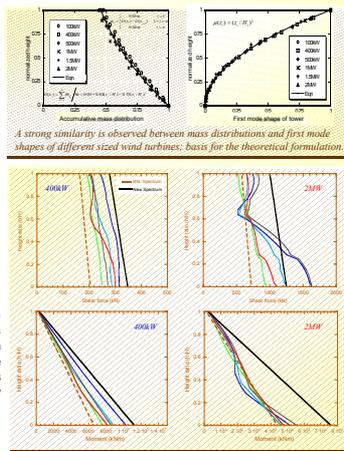
$$\text{Height of gravity centre: } h_c = H_i(0.934 + 0.5C_s)/(1 + C_s)$$

### Modification of present response spectrum

The underestimation of base reactions is caused by large scatter in acceleration spectrum of low damped wind turbine systems as shown. The response spectrum defined by BSL is based on damping ranging 2% to 5% that results in considerable underestimation of the seismic loads. Therefore a modification is introduced to the response spectrum definition, called as safety factor  $\gamma$ , to account for the damping ratio such that:

$$\text{New response spectrum: } S_{aT}(T, \zeta) = \gamma S_{a0}(T, \zeta)$$

where value of  $\gamma$  is 1.7 is used in the present study.



## Conclusions

A full nonlinear FEM model is developed that takes into account the geometric nonlinearity and the coupling between the rotor and tower to perform the time domain analysis of wind turbines. A modal and dynamic response analysis was carried out for level II earthquakes in accordance with new Japanese design code. And semi theoretical formula for estimation of shear and moment profile along the tower height are discussed.

- The contribution of higher modes towards the structural response is small for wind turbines with low natural period. However it becomes important for tall wind turbines resulting in significant overestimation of base moment when WEE model is used.
- Present response spectrum used defined for building structures (BSL, 2004) in Japan could not capture the characteristics of acceleration response spectrum with very low damping such as wind turbines. A modification response spectrum is required current to account for large response of low damped systems.
- The semi-theoretical formula when used along with present response spectrum, defined for buildings, underestimates the seismic loads acting on the wind turbines. A safety factor is introduced to account for the low structural damping and the estimated base loads agrees well with base moment when modified response spectrum is used.
- Base shear is still underestimated by modified spectrum but moment being the main design parameters ensure the usefulness of proposed semi-theoretical formula. Further investigations on the safety factor are needed to accurately predict the seismic loads acting on wind turbines.

### References

- BSL (2004), "The Building standard law of Japan", The building centre of Japan, (both in English and Japanese)
- JSCE (2007), "Guidelines for design of wind turbine support structures and foundations", Japan society of civil engineers (in Japanese)
- Bosanyi, E.A. (2005), "GH Bladed theory manual", Garad Hassan and Partners Limited