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Extreme Wave Height Estimation Formula for a Substructure of an Offshore Wind Turbine

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ABSTRACT

Extreme wave height for design of an offshore wind turbine installed in shallow water zone is calculated based on Goda's theoretical wave deformation model (Goda, 1975; Goda, 2010) in accordance with requirement of exceedance probability specified in IEC61400-3 (2009). Based on the results, Goda's approximation formula for the maximum wave height is adjusted to be consistent with IEC61400-3. Comparisons are made between Battjes & Groenendijk's model and Goda's model. Furthermore, approximation formula for the incipient depth for breaking wave is presented.

KEY WORDS: Extreme wave height; Battjes & Groenendijk model; Goda model; incipient breaking depth; IEC61400-3.

INTRODUCTION

In IEC61400-3 (2009), extreme wave height (EWH) is specified to calculate based on the long term metocean data. However, when the sufficiently long data are not available, it can be calculated using Eq. 1 by assuming the Rayleigh distribution on the water surface motion,

$$H_{T_R} = 1.86 H_{sT_R} \tag{1}$$

where H_{T_R} and H_{sT_R} are the T_R -year expected value of the EWH and the significant wave height for the 3-hour averaging, respectively. H_{T_R} is a wave height with the exceedance probability of 1/1000, which is approximately equal to the maximum wave height among 1000 individual waves (see, Table A.1).

In the case of deep water where wave shoaling deformation is negligible, it is reasonable to estimate EWH by assuming the Rayleigh distribution, thus Eq. 1 can be used. However, as the wave propagates to shorelines, wave height is amplified by the shoaling effect and at the same time the higher portion of wave height distribution is removed by wave breaking and is transformed into the lower portion, consequently wave height distribution deviates from the Rayleigh distribution. Therefore the Battjes & Groenendijk model (Groenendijk and van Gent, 1998; Battjes and Groenendijk, 2000) and the Goda model (Goda, 1975; Goda, 2010) are provided in the IEC standard and JSCE guideline (2010), respectively.

On the other hand, as DNVGL-ST-0437 (2016) and JIS C 1400-3 (2014) have pointed out, BG model has a drawback that it was optimized with a few flume experiment data and was not validated against field measurement data. In DNV, for the usage of the BG model, validation by site-specific wave data is required. Since the BG model is inherently a mathematical fitting by experimental data, wave shoaling and wave breaking which are essential for wave deformation in shallow water zone are considered implicitly through data which were used for curve fitting, however, the Goda model considers these theoretically.

Showing EWH formula consistent with the IEC standard by the Goda model may contribute to the accuracy improvement of EWH calculation. On the other hand, Goda (2012) has made a comparison of his model with BG model and concluded that in intermediate-depth waters BG model underestimates EWH, however, he did not lead the conclusion by showing EWH by his model. Based on the above background, in this paper, estimation of EWH by Goda model is presented and comparison between two models is made.

WAVE HEIGHT ESTIMATION MODEL

Battjes & Groenendijk Model

BG model is a composite Weibull distribution of wave height in shallow water zone with a constant seabed slope as Eq. 2,

$$F_{\underline{H}}(H) \equiv \Pr\{\underline{H} \le H\} = \begin{cases} F_1(H) = 1 - \exp\left[-\left(\dfrac{H}{H_1}\right)^{k_1}\right] & H \le H_{tr} \\ F_2(H) = 1 - \exp\left[-\left(\dfrac{H}{H_2}\right)^{k_2}\right] & H > H_{tr} \end{cases} \tag{2}$$

where, H_{tr} is a transitional wave height, H_1 and H_2 are scale parameters. k_1 and k_2 are $k_1 = 2\left(H \le H_{tr}\right)$ $k_2 = 3.6\left(H > H_{tr}\right)$, respectively, where $k_1 = 2$ represents Rayleigh distribution. Root mean square of wave height H_{rms} and transitional wave height H_{tr} are approximated by Eq. 3 and Eq. 4, respectively, which are obtained empirically from the site sea depth h, seabed slope $\tan\theta$ and standard deviation s of sea surface elevation σ_ζ, based on water flume tests.

$$H_{rms} = \left(2.69 + 3.24\sigma_\zeta / h\right)\sigma_\zeta \tag{3}$$

$$H_{tr} = \left(0.35 + 5.8\tan\theta\right)h \tag{4}$$

Scale parameters H_1 and H_2 are obtained by solving Eq. 5,

$$\begin{cases} \left(\dfrac{\tilde{H}_{tr}}{\tilde{H}_1}\right)^2 = \left(\dfrac{\tilde{H}_{tr}}{\tilde{H}_2}\right)^{3.6} \\ 1 = \sqrt{\tilde{H}_1^2\gamma_1\left[2,\left(\dfrac{\tilde{H}_{tr}}{\tilde{H}_1}\right)^2\right] + \tilde{H}_2^2\gamma_2\left[\dfrac{2}{3.6}+1,\left(\dfrac{\tilde{H}_{tr}}{\tilde{H}_2}\right)^{3.6}\right]} \end{cases} \tag{5}$$

where, $\tilde{H}_{tr} = H_{tr}/H_{rms}$, $\tilde{H}_1 = H_1/H_{rms}$ and $\tilde{H}_2 = H_2/H_{rms}$. $\gamma_1(a,x)$ and $\gamma_2(a,x)$ are the lower and upper incomplete gamma function, respectively.

Normalized wave height of $1/N$ exceedance probability $\tilde{H}_{1/N} = H_{1/N}/H_{rms}$ is determined as follows,

$$\tilde{H}_{1/N} = \tilde{H}_{1/N,1}\ \left(\tilde{H}_{tr} > \tilde{H}_{1/N}\right) \tag{6}$$

$$\tilde{H}_{1/N} = \tilde{H}_2\left[\ln(N)\right]^{\frac{1}{k_2}}\ \left(\tilde{H}_{tr} \le \tilde{H}_{1/N}\right) \tag{7}$$

where,

$$\tilde{H}_{1/N,1} = \tilde{H}_1\left[\ln(N)\right]^{\frac{1}{k_1}} \tag{8}$$

is a normalized wave height of $1/N$ exceedance probability, which is determined according to the Rayleigh distribution.

Normalized $1/N$ maximum wave height $\tilde{\bar{H}}_{1/N} = \bar{H}_{1/N}/H_{rms}$ is also calculated as follows,

for $\tilde{H}_{tr} > \tilde{H}_{1/N,1}$,

$$\begin{aligned} \tilde{\bar{H}}_{1/N} &= N\int_{\tilde{H}_{1/N}}^{\tilde{H}_{tr}} \tilde{H}f_1\left(\tilde{H}\right)d\tilde{H} + N\int_{\tilde{H}_{tr}}^{\infty} \tilde{H}f_2\left(\tilde{H}\right)d\tilde{H} \\ &= N\tilde{H}_1\left(\gamma_2\left[\frac{1}{k_1}+1,\ln(N)\right] - \gamma_2\left[\frac{1}{k_1}+1,\left(\frac{\tilde{H}_{tr}}{\tilde{H}_1}\right)^{k_1}\right]\right) \\ &\quad + N\tilde{H}_2\gamma_2\left[\frac{1}{k_2}+1,\left(\frac{\tilde{H}_{tr}}{\tilde{H}_2}\right)^{k_2}\right] \end{aligned} \tag{9}$$

for $\tilde{H}_{tr} \le \tilde{H}_{1/N,1}$,

$$\begin{aligned} \tilde{\bar{H}}_{1/N} &= N\int_{\tilde{H}_{1/N}}^{\infty} \tilde{H}f_2\left(\tilde{H}\right)d\tilde{H} \\ &= N\tilde{H}_2\gamma_2\left[\frac{1}{k_2}+1,\ln(N)\right] \end{aligned} \tag{10}$$

where, $\tilde{H}_{1/N,1} = \tilde{H}_{1/N}$. f_1 and f_2 are probability density function $f_i(H) = dF_i(H)/dH\ (i = 1,2)$, respectively.

In Groenendijk and van Gent (1998), Battjes and Groenendijk (2000) and Annex C of IEC61400-3 (2009), $H_{1/N}$ ($N = 50$, 100 and 1000) and $\bar{H}_{1/N}$ $\left(N = 3 \text{ and } 10\right)$ are tabulated for convenience.

Goda Model

Wave height in shallow water zone with a constant seabed slope can be estimated by theoretical random wave breaking model proposed by Goda (1975, 2010).

Goda model also reflects effects of wave setup and setdown due to radiation stress and water level fluctuation due to surf beats.

Fig. 1 represents probability density distribution of wave height. In the figures, H, H_{b1} and H_{b2} denote wave height and limiting breaker heights which are defined by Eq. 11, respectively.

$$H_b = AL_0\left\{1 - \exp\left[-1.5\frac{\pi h}{L_0}\times\left(1 + K\tan^s\theta\right)\right]\right\} \tag{11}$$

where, $A = 0.18\left(H = H_{b1}\right)$, $A = 0.12\left(H = H_{b2}\right)$, $K = 15$, $s = 4/3$ and L_0 is wave length at deep water.

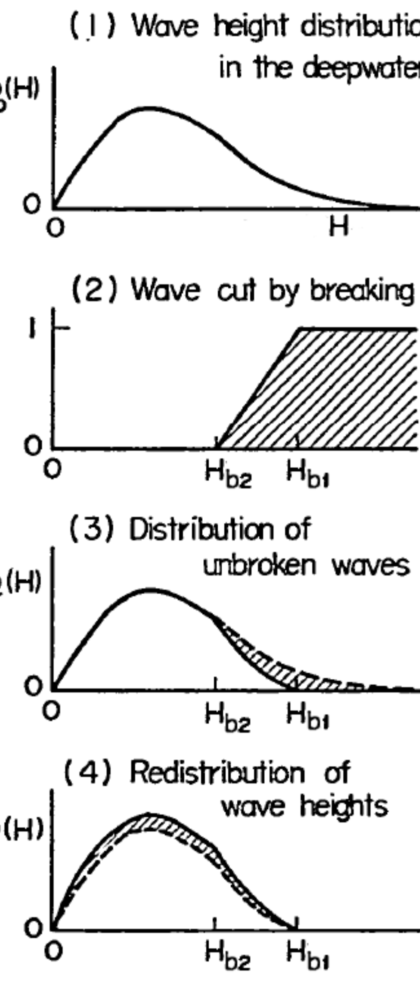


Fig. 1 Explanatory sketch of the model of random waves breaking (Goda, 1975; Goda, 2010)

First, the distribution before wave breaking is assumed to have a Rayleigh distribution as shown in Fig. 1(1). When the wave propagate to shorelines, among the waves obeying that distribution, those with height exceeding the breaking limit H_{b1} will break and cannot occupy their original position in the wave height distribution. Since the wave breaking takes places over a wave height range from H_{b2} to H_{b1} , in this range the wave height distribution is assumed to vary linearly (Fig. 1(2)). As a result, non-breaking portion of the wave height distribution becomes $p_r\left(H\right)$ in Fig. 1(3). The broken waves do not lose all of their energy but retain some. They are assumed to be distributed in the range of non-dimensional wave heights between 0 and H_{b1} with a probability proportional to the distribution of unbroken waves. With this model, the wave height distribution within the surf zone is expressed as $p\left(H\right)$ shown in Fig.1(4).

Within the shallow water zone, the mean water level varies locally due to radiation stress. The variation in the mean water level modifies the local water depth, which determines the breaker height. Because the individual wave heights are controlled by the breaker height, the variations in the mean water level and the wave height distribution must be solved simultaneously. In the Goda model, wave setup and wave setdown is evaluated by solving differential equation with regard to local mean water level from still water level (Longuet-Higgins and Stewart, 1962) by finite difference method.

Another source contributing to the variation in the mean water level is the phenomenon of surf beats. In the Goda model, variation of mean water level due to surf beat is assumed to be normal distribution, and its influence on wave height distribution is evaluated by sum of probability densities of eight representative water depths.

In Fig.2, reproduced significant wave height $\overline{H}_{1/3}$ and $N=250$ maximum wave height $\overline{H}_{1/250}$ by the Goda model are validated against original Goda's calculation results for sea bottom slopes of $\tan\theta=1/10$ and $\tan\theta=1/100$. In Fig.3, variation of mean water level is presented. Hereafter in this paper, these numerical calculations will be called "rigorous" solution of Goda model. In Fig. 2a and Fig. 2c, incipient breaking depths are also presented, they are discussed later.

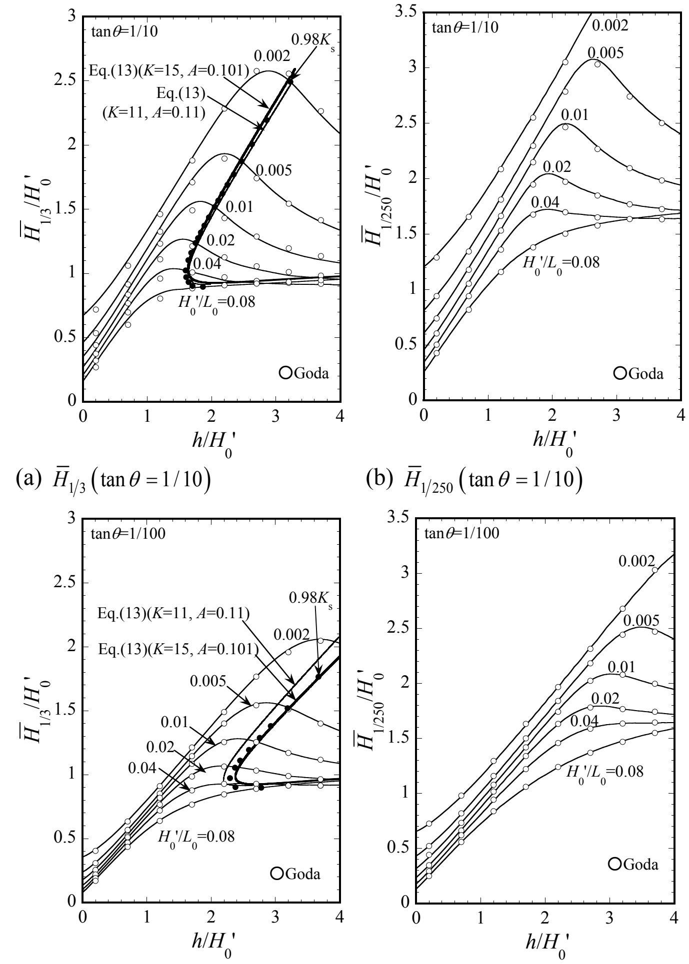
THE EXTREME WAVE HEIGHT BY GODA MODEL

The Extreme Wave Height by Rigorous Calculation

Both BG and Goda models can obtain EWH of arbitrary exceedance probability. In the IEC standards, $H_{1/1000}$ wave height which corresponds to the exceedance probability of 1/1000 is defined as the EWH.

Unfortunately only $\overline{H}_{1/3}$ and $\overline{H}_{1/250}$ are presented in the previously issued literatures by Goda (1975, 2010). Goda (2012) has pointed out that when $1.03\overline{H}_{1/250}$ is compared with $H_{1/1000}$ by BG model, where 1.03 is a ratio between $H_{1/1000}$ and $\overline{H}_{1/250}$ in deep water by assuming Rayleigh distribution, in intermediate-depth waters BG model underestimates $H_{1/1000}$ than Goda model, although he has not showed $H_{1/1000}$ explicitly by his own model in the paper. As EWH is the most important parameter in the design of substructure of offshore wind turbine, in this paper rigorous EWH $H_{1/1000}$ is presented according to Goda model.

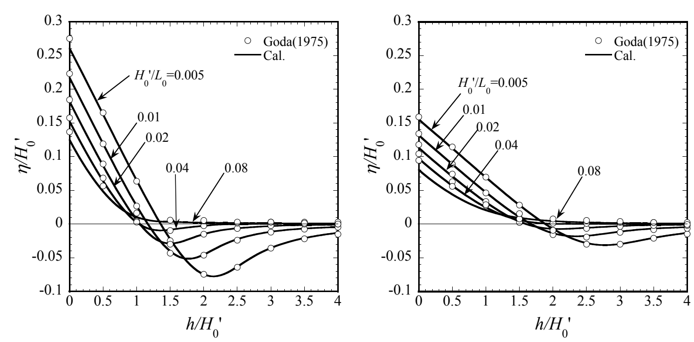
In Fig.4, EWH $H_{1/1000}$ is presented together with 1/400 maximum wave heights $\overline{H}_{1/400}$ and the 1/1000 maximum wave heights $\overline{H}_{1/1000}$. It can be found that even in the shallow water region where the wave breakings plays important roles, $\overline{H}_{1/400}\approx H_{1/1000}$ holds like deep water state which is governed by the Rayleigh distribution, see Table A.1. In addition, $\overline{H}_{1/1000}$ is conservative than $H_{1/1000}$ as expected from the definition.



(a) $\overline{H}_{1/3}\left(\tan\theta=1/10\right)$ (b) $\overline{H}_{1/250}\left(\tan\theta=1/10\right)$

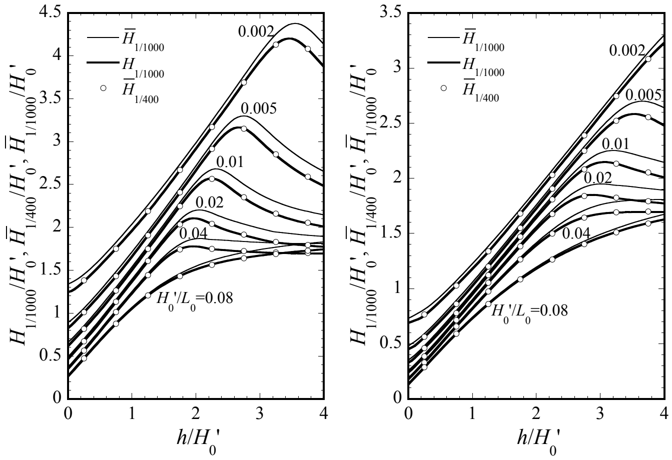
(c) $\overline{H}_{1/3}\left(\tan\theta=1/100\right)$ (d) $\overline{H}_{1/250}\left(\tan\theta=1/100\right)$

Fig. 2 Verification of the analysis ($\overline{H}_{1/3}$ and $\overline{H}_{1/250}$).



(a) $\tan\theta=1/10$ (b) $\tan\theta=1/100$

Fig. 3 Verification of the analysis (Spatial variation of mean water level).



(a) $\tan\theta = 1/10$ (b) $\tan\theta = 1/100$

Fig. 4 Goda's rigorous solutions of $\overline{H}_{1/400}$, $\overline{H}_{1/1000}$ and $H_{1/1000}$.

The Approximation Formula of the Extreme Wave Height

Goda approximated $\overline{H}_{1/N}$ as Eq. 12a,

$$\frac{\overline{H}_{1/N}}{H_0'} \begin{cases} \approx \underbrace{\beta_{K_s} K_s}_{\text{III}} & : h/L_0 \geq 0.2 \\ \approx \min[\underbrace{\left(\beta_0 + \beta_1 \dfrac{h}{H_0'}\right)}_{\text{I}}, & : h/L_0 < 0.2 \\ \qquad \underbrace{\beta_{\max}}_{\text{II}}, \underbrace{\beta_{K_s} K_s}_{\text{III}}] & \end{cases} \tag{12a}$$

Here parameters for $N = 250$ are as follows,

$$\begin{aligned} &\beta_0 = 0.052\left(H_0'/L_0\right)^{-0.38} \exp\left(20\tan^{1.5}\theta\right) \\ &\beta_1 = 0.63\exp\left(3.8\tan\theta\right) \\ &\beta_{\max} = \max\left[1.65, 0.53\left(H_0'/L_0\right)^{-0.29}\exp\left(2.4\tan\theta\right)\right] \\ &\beta_{K_s} = 1.8 \end{aligned} \tag{12b}$$

where $H_0' = K_r K_d \overline{H}_{1/3}$, K_r is refraction coefficient, K_d is diffraction coefficient, L_0 is deep water wave length and K_s is shoaling coefficient. The above formula can be illustrated as Fig.5, where wave heights in shallow water zone is expressed in terms of two straight lines I and II which correspond to wave breaking and a curve III which shows shoaling. β_0 and β_1 are y-axis intercept and slope of the linear line I, respectively, $\beta_{\max}$ is a peak wave height by linear line II.

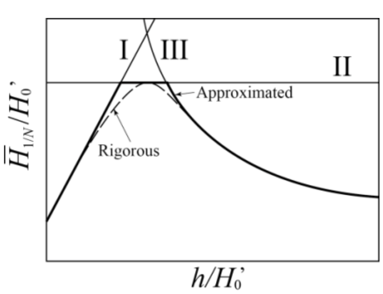


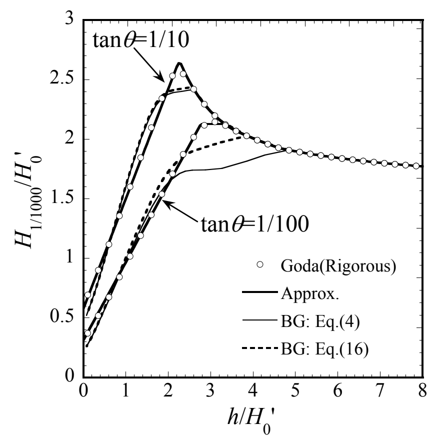
Fig. 5 Explanatory sketch of Eq. 12a.

Following the formulas of Eq. 12a, approximate formulas for EWH $H_{1/1000}$ is written as follows,

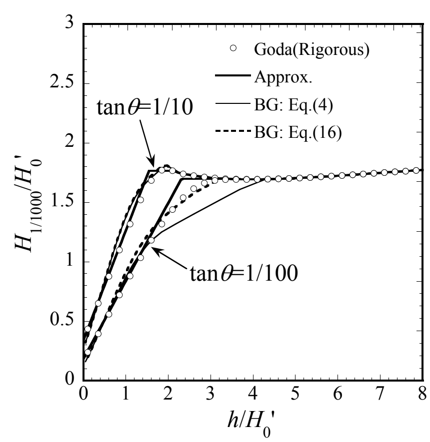
$$\frac{H_{1/1000}}{H_0'} \begin{cases} \approx 1.03 \times 1.8 K_s & : h/L_0 \geq 0.2 \\ \approx \min[\left(1.03\beta_0 + \beta_1 \dfrac{h}{H_0'}\right), & : h/L_0 < 0.2 \\ \qquad 1.03\beta_{\max}, 1.03 \times 1.8 K_s] & \end{cases} \tag{12c}$$

where, β_0 , $\beta_{\max}$ and β_{K_s} are multiplied by a factor 1.03 which is obtained in deep water relation of $H_{1/1000}/\overline{H}_{1/250} = 1.86/1.8 \approx 1.03$, see Table A.1. In Fig. 6, comparisons are made between rigorous and approximated $H_{1/1000}$. Although in those figures, comparisons with BG model are also presented, it will be discussed later.

If $K_r = K_d = 1$, in Fig. 6, $H_0'/L_0 = 0.01$ and $H_0'/L_0 = 0.04$ are representative sea states for wind generated wave and swell, respectively, see e.g. Goda (2010). From the Fig. 6a, it can be found that $H_{1/1000}$ is well approximated for both $\tan\theta = 1/10$ and $\tan\theta = 1/100$. On the other hand, in Fig. 6b, when the seabed slope becomes mild, at a transition region from I to II in Fig.5, the approximation overestimates the rigorous solution, however, this is a limit of bilinear approximation as Goda (2010) also pointed out.



(a) $H_0'/L_0 = 0.01$



(b) $H_0'/L_0 = 0.04$

Fig. 6 Comparisons between rigorous and approximated $H_{1/1000}$.

Usage of the approximation by Eq. 12c is limited dependent on the excess probability of wave height. Fig. 7 shows $H_{1/1000}$ against h/L_0 together with shoaling coefficient K_s . In the case of $h/L_0 < 0.2$, bilinear approximation is found to be valid for $H_0'/L_0 \le 0.05$. Therefore in the case of $H_{1/1000}$, approximation by Eq.12c is valid for $H_0'/L_0 \le 0.05$.

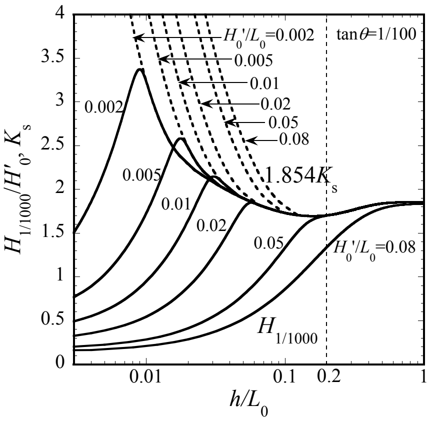


Fig. 7 Relation of $H_{1/1000}$ and K_s versus h/L_0 $(\tan\theta = 1/100)$

Constrained Wave

In IEC 61400-3 (2009), simulation which uses EWH is required to use water particle motion which consider the nonlinearity of wave kinematics, a constrained wave method, see e.g. Taylor (1997), is recommended. A constrained wave method is an analytical technique which embeds a regular nonlinear wave motion which corresponds to desired EWH into a background linear random wave motion to enhance the reduction of amount of a simulation.

Although any kinds of method are available for a generation of nonlinear regular wave, a favorable candidate is a one which can take into account of i) seabed slope and ii) wave amplification due to wave setup and surf beat near shoreline. However, such kinds of technique are not common to practical design.

The stream function method is a common method which can be used in the constrained wave method to generate nonlinear regular wave, however, since it is developed for wave motion with flat bottom seabed, i.e. $\tan\theta = 0$, therefore i) is not satisfied. Furthermore, ii) is satisfied with, neither. However, stream function method is favorable to design because of its easiness of use. If one faced against these cases, although it may be expedient, one should generate EWH by adjusting water depth to match the desired EWH, subsequently the water particle motion is adjusted by a certain stretching method.

Incipient Breaking Depth

There are few studies on a depth at which breaking wave impact force should be taken into account. One idea is to employ Goda's incipient breaking depth as an indication, and breaking wave impact force is taken into account some ways in a shallower depth than the depth. The incipient breaking depth $\overline{h}_{-2\%}$ is defined as a depth where wave height is 2% smaller than the wave height which is estimated by shoaling effect only, and is obtained by solving Eq. 11 with respect to water depth after replacing its left hand side by $0.98 K_s H_0'$.

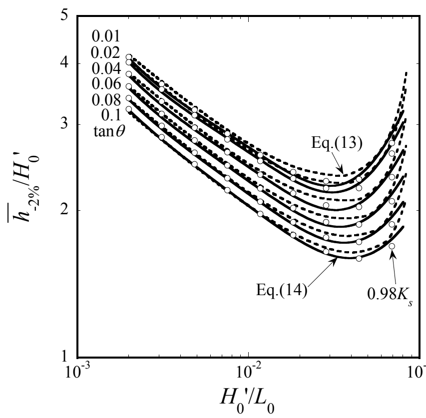
$$\overline{h}_{-2\%} = -\frac{L_0}{1.5\pi\left(1 + K\tan^{4/3}\theta\right)}\ln\left[1 - \frac{0.98 K_s H_0'/L_0}{A}\right] \tag{13}$$

Eq. 13 needs to be solved iteratively with respect to water depth, since K_s is also a function of water depth.

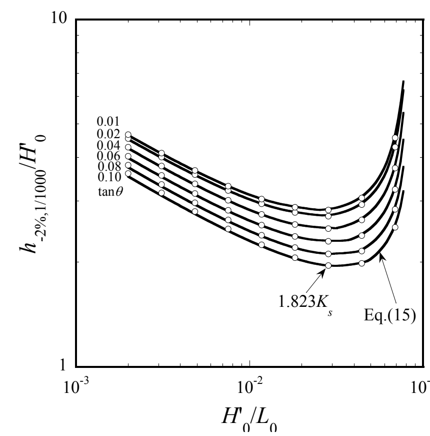
Goda (2010) suggested to use $A \approx 0.11$ when the incipient depth is calculated. In Fig. 2, the incipient depth which is obtained by solving Eq. 13 with $A \approx 0.11$ is plotted, however, it is underestimated in case of $\tan\theta = 1/100$. Therefore the parameter A was adjusted to fit with both of $\tan\theta = 1/10$ and $1/100$. Here, $A = 0.101$ is found to be a better choice and to fit well with both of the seabed slopes.

Goda approximated the water depth for peak significant wave height, however, did not give an approximation of incipient breaking depth. Eq. 14 is an approximation of the incipient breaking depth for $\overline{H}_{1/3}/H_0' = 0.98K_s$. Here, $x = \ln\left(H_0'/L_0\right)$ and $y = \ln\left(\tan\theta\right)$, and is applicable to $0.002 \le H_0'/L_0 \le 0.08$ and $1/100 \le \tan\theta \le 1/10$. In Fig.8a, $\overline{h}_{-2\%}$ by Eq. 13 and Eq. 14 are compared with $0.98K_s$. Either formula can be used to approximate $\overline{h}_{-2\%}$.

$$\begin{aligned} \overline{h}_{-2\%}/H_0' &= \sum_{i=0}^{4} a_i x^i \\ a_0 &= -36.15 - 27.474y - 2.8243y^2 \\ a_1 &= -30.9 - 22.916y - 2.3567y^2 \\ a_2 &= -9.8773 - 7.3272y - 0.75418y^2 \\ a_3 &= -1.3948 - 1.0289y - 0.10591y^2 \\ a_4 &= -0.07286 - 0.053969y - 0.0055565y^2 \end{aligned} \tag{14}$$



(a) For significant wave height



(b) For EWH

Fig.8 Incipient breaking depth

Similarly, incipient breaking depth for EWH can be formulated in the same manner as Eq. 13. It can be calculated by solving Eq. 15 iteratively with respect to water depth, however, here with $A = 0.152$. In Fig. 8b, it is compared with $0.98 \cdot 1.86 K_s$.

$$h_{-2\%,1/1000} = -\frac{L_0}{1.5\pi\left(1 + K\tan^{4/3}\theta\right)} \ln\left(1 - \frac{0.98 \cdot 1.86 K_s H_0'/L_0}{A}\right) \quad (15)$$

COMPARISON OF THE EXTREME WAVE HEIGHT PREDICTION MODEL

Common input parameters of the BG model and Goda model are water depth h and sea bed slope $\tan\theta$. In addition to these parameters variance of the water surface motion σ_ζ^2 and the equivalent deep water wave height H_0' is necessary for the BG model and the Goda model, respectively, which prevents direct comparison of the models. Therefore, for the comparison, σ_ζ has to be related with H_0'. It should be reminded that if the target site locates inside shallow water zone influence of breaking wave is included in σ_ζ. Therefore a relationship in IEC61400-3 of $H_0 = 0.956 H_{s0}\left(H_{s0} = 4\sigma_{\zeta_0}\right)$ which is valid in the deep water where the Rayleigh distribution assumption is applicable cannot be applied to relate σ_ζ with H_0'. BG model approximates H_{rms} by Eq. 3 in terms of σ_ζ, however, Goda model can also yield H_{rms}. Here, H_{rms} is yielded according to Goda (2012), however, detailed procedures are not described in the literature. In this paper, comparisons of EWH are made as follows, see Fig. 9. At first, a series of curves of H_{rms}/H_0' are calculated for each H_0'/L_0 with respect to specific sea bottom slope by Goda's rigorous model as Fig.10a. Next, H_{rms} is calculated by assuming Eq. 3. Abscissa value ξ at the intersection of a line by H_{rms}/h with these curves will yield h/H_0' which corresponds to input parameter σ_ζ^2. H_0' is calculated as $H_0' = h/\xi$ and consequently wave height which corresponds to ξ in Fig.10b will be EWH which corresponds to the BG model. Since H_{rms} is dependent on H_0'/L_0, it is a function of unknown H_0', however, because here a comparison between both models is the purpose, H_{rms} are calculated in advance for known values of $0.002 \le H_0'/L_0 \le 0.08$.

In the preceding Fig.6, comparisons are made for $H_{1/1000}$ between the BG model and the Goda model. Both models are well corresponded overall except that original BG model tends to underestimate peaks which are expressed by a line II in Eq. 12, and it is remarkable as the seabed slope $\tan\theta$ becomes smaller. In the figure, another BG model where transitional wave height is modified as Eq. 16 is presented by dashed line.

$$H_{tr} = \left(0.5 + 4.5\tan\theta\right)h \quad (16)$$

Underestimation is improved especially in the case of $\tan\theta$ is small.

In Fig.11, a comparison is made between both of the models with respect to an example in IEC61400-3 (2009) where $\sigma_\zeta^2 = 1.1\times10^{-3}$ m, $h = 0.27$m and $\tan\theta = 1/100$. In this example, either wave height distributions of which the exceedance probability is below 40% deviates from the curve according to the Rayleigh distribution, showing wave breakings. When Eq. 16 is used, underestimation of BG model in this region is improved.

In Fig.12, ratios of $H_{1/1000}$ between both of the models are plotted against h/H_{rms} for the different seabed slopes. Although it depends on h/H_{rms}, they coincide each other at $h/H_{rms} = 4$. This is because $h/H_{rms} = 4$ is deep water region where damping effect by wave breaking is insignificant. Looking at the influence of the seabed slope, the underestimation of the BG model is remarkable in the case of the original BG model using the Eq. 4 for the transition wave height when the seabed gradient is small like $\tan\theta = 1/100$. By modifying the transition wave height with Eq. 16, the underestimation of the BG model is improved for $2.5 < h/H_{rms} < 5$ in the case of $\tan\theta = 1/100$.

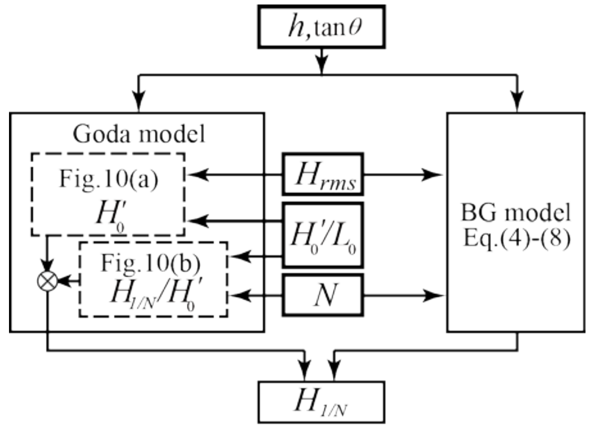
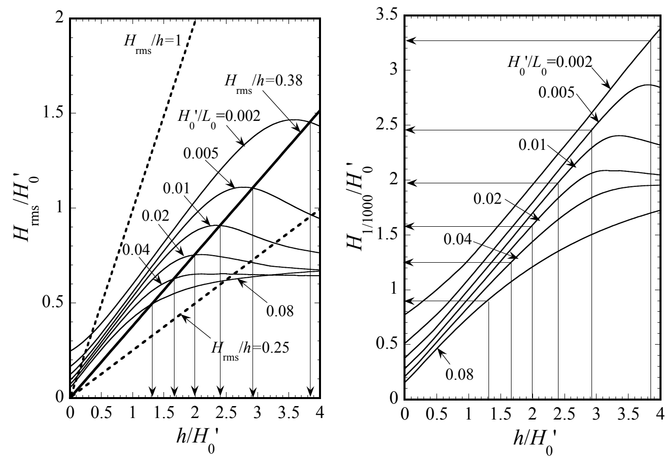


Fig. 9 Explanatory chart of comparison of $H_{1/N}$ by Battjes & Groenendijk model and Goda model.



(a) H_{rms} (b) $H_{1/1000}$

Fig. 10 Solution procedure of $H_{1/1000}$ from H_{rms}/h ($H_{rms}/h = 0.38$, $\tan\theta = 1/100$).

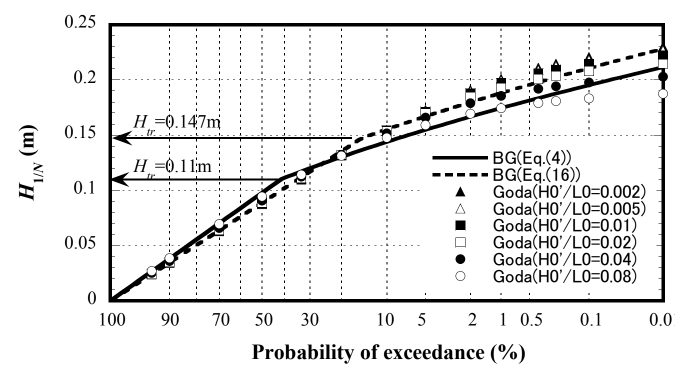
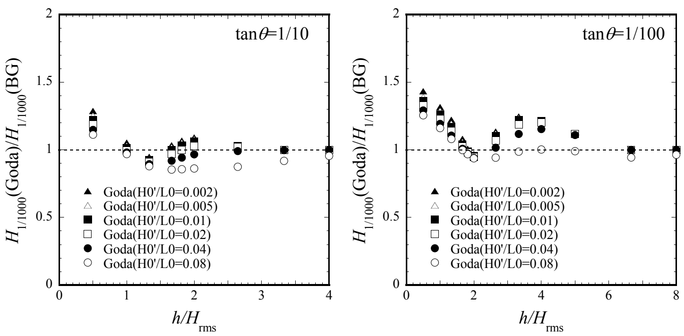
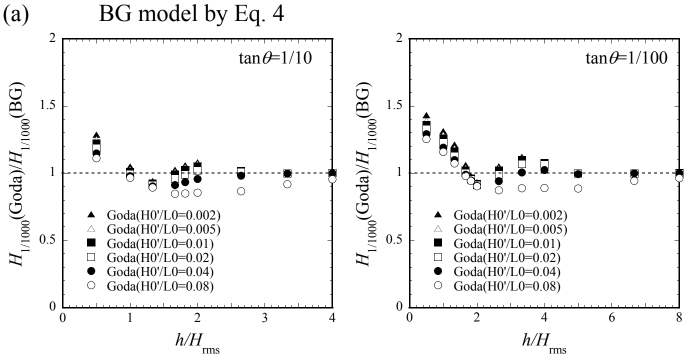


Fig. 11 Comparison of $H_{1/N}$ by Battjes & Groenendijk model and Goda model ($H_{rms}/h = 0.38$, $\tan\theta = 1/100$).



(a) BG model by Eq. 4



(b) BG Model by Eq. 16

Fig. 12 Comparison of $H_{1/1000}$ by Battjes & Groenendijk model and Goda model for different seabed slopes.

CONCLUSIONS

Based on Goda model, with regard to the extreme wave heights which are consistent with the definition in IEC 61400-3 was discussed. The conclusions are summarized as follows,

1. A series of rigorous curves and approximation formula of the extreme wave height which is consistent with the definition of IEC61400-3 was obtained based on the Goda's wave breaking deformation model of random waves for constant seabed slope.

2. Estimation formulae were presented for the incipient breaking depth with respect to both of the significant wave height and the extreme wave height. For the former, an approximation formula was also presented.

Comparisons were made between the Battjes & Groenendijk model and the Goda model. As a result, peaks of the extreme wave height in shallow water zone by the former model were underestimated than by the latter model. However, it can be improved by adjusting the transition wave height as slightly higher.

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APPENDIXES

Derivation of the statistical wave heights under the assumption of the Rayleigh distribution

According to Longuet-Higgins (1952), when the wave height distribution is assumed to follow the Rayleigh distribution, the ratios of the $1/N$ averaged wave height $\overline{H}_{1/N}$, the most probable highest wave height among N_0 individual wave heights $\hat{H}_{\mathrm{mode}}(N_0)$ and the wave height which corresponds to the probability of exceedance of $H_{1/N}$ relative to the statistical significant wave height $\overline{H}_{1/3}$ are expressed as follows,

$$\frac{\overline{H}_{1/N}}{\overline{H}_{1/3}} = \frac{\sqrt{\ln N} + \dfrac{N\sqrt{\pi}}{2}\left\{1 - \mathrm{erf}\left(\sqrt{\ln N}\right)\right\}}{1.416} \tag{17}$$

$$\frac{\hat{H}_{\mathrm{mode}}(N_0)}{\overline{H}_{1/3}} \cong \frac{\sqrt{\ln N_0}}{1.416} \tag{18}$$

$$\frac{H_{1/N}}{\overline{H}_{1/3}} = \frac{\sqrt{\ln N}}{1.416} \tag{19}$$

where, erf is an error function defined as

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt\ .$$

Table A.1 shows those values for typical N and N_0, however, $\hat{H}_{\mathrm{mode}}(N_0)/\overline{H}_{1/3}$ is not approximated value by Eq. A.2 but is obtained by Newton method according to Longuet-Higgins (1952).

Table A.1. Statistical wave heights under the assumption of the Rayleigh distribution

N, N_0	$\overline{H}_{1/N}/\overline{H}_{1/3}$	$H_{1/N}/\overline{H}_{1/3}$	$\hat{H}_{\mathrm{mode}}(N_0)/\overline{H}_{1/3}$
250	1.799	1.659	1.673
400	1.863	1.729	1.741
1000	1.982	1.856	1.866

Proposal of Estimation Formula of the Extreme Wave Height to IEC61400-3

Proposed estimation formula of the extreme wave height is Eq.12c and can be rewrite as follows,

$$\frac{H_{1/1000}}{H_0'}\begin{cases}\approx 1.86K_s & : h/L_0 \geq 0.2\\ \approx \min[\left(\beta_0^* + \beta_1^* \dfrac{h}{H_0'}\right), & : h/L_0 < 0.2\\ \qquad \beta_{max}^*, 1.86K_s] & \end{cases} \tag{20}$$

$$\beta_0^* = 0.054\left(H_0'/L_0\right)^{-0.38}\exp\left(20\tan^{1.5}\theta\right)$$
$$\beta_1^* = 0.63\exp\left(3.8\tan\theta\right) \tag{21}$$
$$\beta_{max}^* = \max\left[1.7, 0.55\left(H_0'/L_0\right)^{-0.29}\exp\left(2.4\tan\theta\right)\right]$$

where, $H_{1/1000}$ is extreme wave height, h is water depth, $\tan\theta$ is seabed slope, K_s is shoaling coefficient and L_0 is wave length at deep water. H_0' is defined as

$$H_0' = K_r K_d H_0 \tag{22}$$

where, K_r is refraction coefficient and K_d is diffraction coefficient. H_0 is statistical significant wave height at deep water which can be obtained as follows if the water surface elevation is assumed to follow the Rayleigh distribution,

$$H_0 = 4.004\sigma_{\zeta_0} \tag{23}$$

Here, σ_{ζ_0} is standard deviation of water surface elevation at deep water. Spectral significant wave height is defined as,

$$H_{s0} = 4\sigma_{\zeta_0} \tag{24}$$

Hence, $H_0 \cong H_{s0}$ is obtained if the surface elevation at deep water is assumed to follow the Rayleigh distribution. Since according to Goda's observation, statistical significant wave height can be estimated as follows (Goda, 2010)

$$H_0 \approx 3.83\sigma_{\zeta_0} \tag{25}$$

thus following relationship can be established between H_0 and H_{s0},

$$H_0 \approx 0.956H_{s0} \tag{26}$$