

Prediction of Wave-induced Tower Loading of Floating Offshore Wind Turbine Systems

Nan Xu and Takeshi Ishihara

Department of Civil Engineering, the University of Tokyo,
Tokyo, Japan

ABSTRACT

In this study, SR (Sway-Rocking) model is proposed to consider the floater surge and pitch motions which have large influence on the tower loading of floating wind turbine, so that the tower loading can be estimated by the equivalent static method. A theoretical comparison of shear force between SR model and fixed-foundation model with floater acceleration acting on the tower base is performed and thus the problems of using the latter model have been clarified. The theoretical formulae to predict the wave-induced tower loading in the extreme wave conditions are proposed finally.

KEY WORDS: SR (Sway-Rocking) model; surge motion; pitch motion; tower loading; complete quadratic combination (CQC) rule; floating offshore wind turbine; extreme wave conditions.

INTRODUCTION

Wind power is one of the fastest growing renewable energy technologies. Onshore wind farms are, however, unsightly and they swallow up valuable land for agriculture and urban development. Already some countries, are considering constructing huge wind farms offshore to take advantage of the generally steadier and stronger winds found in the sea (Wang, 2010). In Japan, the offshore consist of a vast wind resource in deep water where use of conventional bottom-mounted wind turbines is not possible, and floating wind turbines are the most attractive.

Based on the research of Henderson (2000), Jonkman (2007), and Phuc (2008), Syed (2010) developed a fully nonlinear finite element model (FEM) to investigate dynamic response of floating offshore wind turbine systems considering coupling between wind turbine, floater and mooring system. This model has been verified by a water tank experiment, thus can give accurate and realistic prediction of floater motion and tower loading due to wind and wave. With this FEM code the land-based wind turbine is compared under the same wind conditions as considered for the offshore floating wind turbines. It is found that floater motion increases the tower base moment. Thus, it is necessary to consider the effect of floater motion on the tower loading

to check the serviceability of the wind turbines which are designed for the bottom-mounted systems. The problem is that all the previous research is based on numerical simulation, and the wave-induced load and wind-induced load are coupled together and the effects from different degrees of freedom of floater motion are also coupled, as a result the contribution of each motion is unclear yet. In most of the real designs of floating offshore wind turbine, the work of wind engineers and ocean engineers is separated. The ocean engineers can provide the floater response without considering the effect from the superstructure (wind turbine part), and the wind engineers usually only concern the wind load acting on wind turbine. Hence, the connection work for wind engineers to calculate the wave-induced load using the floater motion provided by the ocean engineers needs to be performed. Therefore, it would make sense for the wave-induced load and wind-induced load to be investigated independently, and then their combination can be performed to get the final design value. For each kind of load, the analytical formulae should be proposed to make the application more convenient and identify their dominant influence factors as well, which would be very useful for the optimization of floating wind turbine system. This study will employ the FEM code of Syed (2010) to simulate the floater motion and verify the analytical solution of tower loading.

In order to propose the analytical formulae of wave-induced load, the calculating model is quite important. Takahashi (2006) used the fixed-foundation model with acceleration acting on tower base to consider the influence of floater motion on the fatigue load. However, this fixed-foundation model is not verified, and in most cases it is not able to produce the reasonable results. Hence, in this study it is necessary to propose an equivalent calculating model for wave-induced tower loading of floating wind turbine system.

In this study, it is found that the nonlinearity from nonlinear hydrodynamic force is much more dominant than that from nonlinear mooring system, hence, the sway-rocking model which models the complex nonlinear mooring system of floating wind turbine as two kinds of linear springs and dampers is borrowed from earthquake engineering to clarify the contribution of each motion to the tower loading. Sway (surge motion) can be represented with the lateral spring

and rocking (pitch motion) with rotational spring. The effects of floater motion will be considered by acting a wave force on the floater. Different from earthquake engineering, the stiffness and damping should be identified by free vibration simulation using FEM, and wave force should be determined by known tower base response. Through the comparison with the full model by FEM simulation, SR model is verified to be the equivalent calculating model for wave-induced tower loading of floating wind turbine system. A theoretical comparison of shear force under regular wave by modal analysis is performed between SR model and fixed-foundation model to give a clear explanation why the latter model can not be used. With SR model, the effect of sway motion as well as rocking motion can be recognized separately by locking the other mode. Since the maximum response of sway and rocking can't occur simultaneously, the combination of them becomes important. Referring to the seismic loads specified in AIJ (2004), square root of sum of squares (SRSS) and complete quadratic combination (CQC) are used for the combination. Through comparison, it will be determined that the CQC rule can give much better predictions for floating wind turbine system. Under irregular wave, the standard deviation is an important representative value for the dynamic wave load. It can also be calculated by the combination of sway and rocking effects by CQC rule. Thus, the maximum wave load can be calculated with the product of standard deviation and peak factor. A non-Gaussian peak factor model that mainly results from the resonance of tower vibration will be proposed in this research.

All the described theories in this study are used for DLC6.2a of IEC-61400-3 (2008) which is the extreme state where the wind turbine is in parked condition, so the effect of wind turbine control system is not taken into account here. It is also noted that this study focuses on the wave-induced load, and the wind load on wind turbine can be estimated by SR model as well, which has been discussed by Xu (2013).

FLOATING WIND TURBINE SYSTEMS

This study will use a semi-submersible type floater installed with NREL 5-MW baseline wind turbine with tension leg mooring and catenary mooring to investigate the influence of floater motion on tower loading, respectively. FEM considering coupling between wind turbine, floater and mooring system developed by Syed (2010) will be reviewed briefly. This code is used to simulate the floater motion and verify the analytical solution of tower loading.

Properties of Floating Wind Turbine System

The National Renewable Energy Laboratory's (NREL) offshore 5-MW baseline wind turbine is used here. For detail regards the wind turbine reference made by Jonkman (2007). As the developed program is not able to consider pitch control, wind turbine is considered as stall regulated. The basic properties of this wind turbine are summarized in Table 1. The details of floater are available in the doctoral dissertation of Syed (2010). The salient features of the floater are listed in Table 2.

The tension leg mooring arrangement is shown in Fig. 1 (a). Three tethers are considered, that are connected to each of the corner columns. The mooring arrangement is so considered to eliminate pitching motion of the floater. The catenary mooring system is considered to consist on three mooring lines, each having span of 400 m. The mooring lines are separated at 120°, with front two lines having an angle of 60° with the incident wave and the third aligned in the wave direction. All the three lines have a common fairlead at the base of the central column of the floater that supports the wind turbine on top. The catenary mooring arrangement is shown in Fig. 1 (b).

Table 1. Properties of NREL 5MW wind turbine (Jonkman, 2007)

Rated Power	5 MW
Rotor Orientation, Configuration	Upwind, 3-blades
Rotor, Hub diameter	126, 3 m
Hub Height	90 m
Cut-In, Rated, Cut-Out wind speed	5.0, 11.4, 25 m/sec
Cut-In, Rated Rotor Speed	6.9 rpm, 12.1 rpm
Rated tip speed	80.0 m/sec
OverHang, Tilt	5.0 m, 5.0°
Rotor Mass	110,000 Kg
Tower Mass	240,000 Kg

Table 2. Details of semi-submersible floater (Syed, 2010)

Description	Detail	Dimension
Span		60.0 m
Submerged Depth		20.0 m
Overall Height		30.0 m
Total Weight		5,638,760 Kg
Peripheral Bracing		φ 2.5 m
Inner Bracing	Top	φ 1.8 m
	Inclined	φ 1.8 m
	Bottom	φ 1.8 m
Corner Column	Top	φ 9.0 m
	Bottom	φ 10.0 m
Central Column		φ 9.0 m

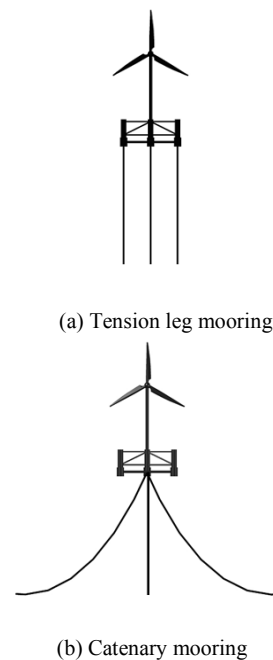


Fig. 1 Types of mooring systems analyzed in this study

Finite Element Model

A finite element model that can use beam, truss and spring type elements and can consider full coupled interaction between wind turbine, floater and mooring system has been developed by Syed (2010). The time domain analysis enables the model to efficiently capture nonlinear effects. Morison equation with Srinivasan's Model is used for estimation of hydrodynamic force on the system, restoring force is investigated using a proposed non-hydrostatic model and mooring force is estimated using nonlinear model considering mooring contact with seabed for catenary mooring and pre-tension for tension leg mooring. For details of this finite element model refers to the doctoral dissertation of Syed (2010), and here a summary of the numerical scheme is presented in Table 3.

Table 3. Description of finite element numerical scheme (Syed, 2010)

Dynamic Analysis	Direct Implicit Integration (Newmark- β)
Formulation	Total Lagrangian formulation
Convergence	Newton-Raphson Method
Damping Estimation	Caughey Series
Element Type	Beam (12-DOF), Truss (8-DOF)
Aerodynamic force	Quasi-static aerodynamic theory
Hydrodynamic Force	Morison Equation + Srinivasan Model
Restoring Force	Non-Hydrostatic Model
Mooring Force	Nonlinear
Seabed contact	Penalty Method

SWAY-ROCKING MODEL

In order to propose the analytical formulae for wave-induced tower loading, an equivalent calculating model of floating wind turbine system is in need. In this study, sway-rocking model shown in Fig. 2 is borrowed from earthquake engineering (AIJ, 2004) to clarify the contribution of each motion to the tower loading. The complex mooring system of floating wind turbine system is modeled as two kinds of springs and dampers. Sway (surge motion) can be represented with the lateral spring and rocking (pitch motion) with rotational spring. The effects of floater motion will be considered by acting a wave force on the floater. Different from earthquake engineering, the stiffness and damping should be identified by free vibration simulation using FEM, and wave force should be determined by known tower base response.

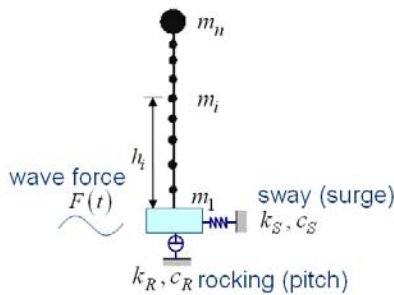


Fig. 2 Sway-rocking model

In order to give a clear explanation about the tower loading, the wind turbine (three blades, hub, nacelle, and tower) can be modeled as 11

lumped masses (Table 4), since the aerodynamic force is not considered here. The three blades are regarded as rigid approximately and can be modeled as a large mass above the tower top with hub and nacelle together. The tower is divided into ten masses.

Table 4. Lumped mass of wind turbine

Height from tower base h_i (m)	Lumped mass m_i (kg)
0	4134403.52
8.76	45861.86
17.52	42825.06
26.28	39891.40
35.04	37060.89
43.80	34333.51
52.56	31709.23
61.32	29188.10
70.08	26770.12
78.84	24455.25
87.60	361661.80

Stiffness and Damping

With the full model of floating wind turbine system, taking the superstructure (wind turbine and floater) as rigid body, the sway frequency ω_S and rocking frequency ω_R can be obtained by free vibration simulation using FEM. Thus, the stiffness of the two springs can be calculated as follows:

$$k_S = \left(\sum_{i=1}^n m_i \right) \omega_S^2 \quad (1)$$

$$k_R = \left(\sum_{i=1}^n m_i h_i^2 \right) \omega_R^2 \quad (2)$$

The sway damping ratio ξ_S and rocking damping ratio ξ_R can also be estimated by comparing with the full dynamic FEM program.

Equivalent Wave Force

Since the tower base response can be known from the ocean engineer in the real project, which means the displacement $[x_1]$, velocity $[v_1]$ and acceleration $[a_1]$ at the tower base are given. In this study, the tower base response can be obtained from simulation. With modal analysis, the equivalent wave force can be calculated.

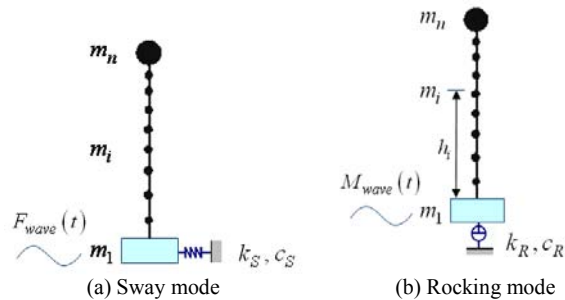


Fig. 3 Calculating model used in the modal analysis

By locking the rocking mode as shown in Fig. 3 (a), the modal equation of motion of j th mode in sway direction is:

$$M_j^S \ddot{f}_j^S(t) + C_j^S \dot{f}_j^S(t) + M_j^S \omega_j^{S2} f_j^S(t) = \begin{bmatrix} \phi_{nj}^S \\ \vdots \\ \phi_{1j}^S \end{bmatrix}^T \begin{bmatrix} 0 \\ \vdots \\ F_{wave}^S(t) \end{bmatrix} \quad (3)$$

$$= \phi_{1j}^S F_{wave}^S(t) \quad (j=1, \dots, n)$$

where $M_j^S = \sum_{k=1}^n m_k \phi_{kj}^{S2}$,

$$C_j^S = \sum_{k=1}^n c_k \phi_{kj}^{S2} \quad (c_1 = c_S)$$

M_j^S is the generalized mass, C_j^S is the generalized damping and ω_j^S is the modal natural frequency in radians per second, f_j^S is the modal displacement, ϕ_{kj}^S ($k=1, \dots, n$) is the normalized mode shape of the j th mode, and $F_{wave}^S(t)$ is the equivalent wave force in sway direction. If the regular wave is used, the modal displacement $f_j^S(t)$ can be shown as:

$$f_j^S(t) = \phi_{1j}^S F_{wave}^S(t) |H_j^S(\omega)| \quad (4)$$

where $|H_j^S(\omega)| = \frac{1}{M_j^S \omega_j^{S2} \sqrt{(1 - \beta_j^{S2})^2 + 4\xi_j^{S2} \beta_j^{S2}}}$

$$\beta_j^S = \frac{\omega}{\omega_j^S}$$

$$\xi_j^S = \frac{C_j^S}{2M_j^S \omega_j^S}$$

β_j^S is the ratio between external wave frequency ω and structural natural frequency, and ξ_j^S is the damping ratio, which is taken as the summation of structural damping ratio and hydrodynamic damping ratio.

In modal analysis the excitations of the various different natural modes of vibration are computed separately and the results superposed. From Eq. 4, the tower base displacement can be calculated as:

$$x_1^S(t) = \sum_{j=1}^n f_j^S(t) \phi_{1j}^S = F_{wave}^S(t) \sum_{j=1}^n |H_j^S(\omega)| \phi_{1j}^{S2} \quad (5)$$

Hence, the equivalent wave force in sway direction can be calculated as:

$$F_{wave}^S(t) = \frac{x_1^S(t)}{\sum_{j=1}^n |H_j^S(\omega)| \phi_{1j}^{S2}} \quad (6)$$

The equivalent wave moment in rocking direction can be calculated by locking the sway mode as shown in Fig. 3 (b) and using the modal analysis as well:

$$M_{wave}^R(t) = \frac{\theta_1^R(t)}{\sum_{j=1}^n |H_j^R(\omega)| \phi_{1j}^{R2}} \quad (7)$$

$\theta_1^R(t)$ is the tower base angular displacement, $|H_j^R(\omega)|$ is the frequency response function, and ϕ_{kj}^R ($k=1, \dots, n$) is the normalized mode shape of the j th mode in rocking direction.

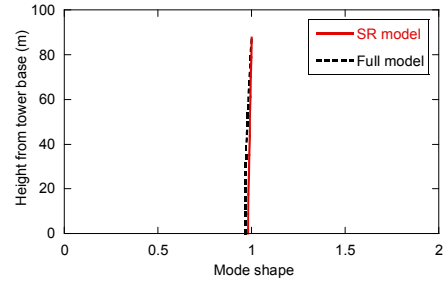
From Eqs. 6~7, the equivalent wave force or moment can be calculated with the tower base displacement, damping ratio, the ratio between external wave frequency and structural natural frequency, and the mode shape of tower base.

Verification of Sway-Rocking Model

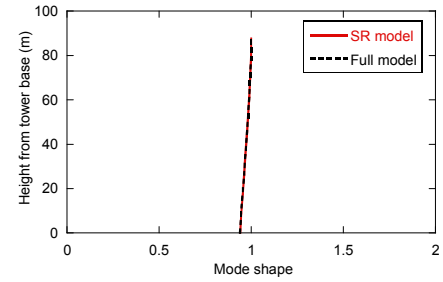
The natural periods of the two kinds of floating wind turbine system are tabulated in Table 5. The first mode shape is shown in Fig. 4. It is noticed that the sway-rocking model is able to give very close natural periods and mode shape to the full model. Fig. 5 shows the comparison of the shear force on wind turbine tower by FEM simulation. It is obvious that sway-rocking model shows good agreement with full model. Therefore, sway-rocking model is verified as the equivalent model to calculate the wave-induced tower loading for floating wind turbine system.

Table 5. The first natural periods

	Tension leg system Full model / SR model	Catenary system Full model / SR model
Sway	31.3 s / 31.9s	26.8s / 26.2s
Rocking	-	14.3s / 15.0s



(a) Sway of tension leg system



(b) Rocking of catenary system

Fig. 4 The first mode shape of wind turbine tower

COMPARISON BETWEEN SR MODEL AND FIXED-FOUNDATION MODEL

A theoretical comparison between sway-rocking model and fixed-foundation model (Takahashi, 2006) is performed to make their difference clear. The shear force at different tower height is derived with modal analysis for the sway-rocking model and fixed-foundation model. For the two models, since the first mode is dominant, the shear force from the first mode is compared.

Shear Force of Sway-Rocking Model

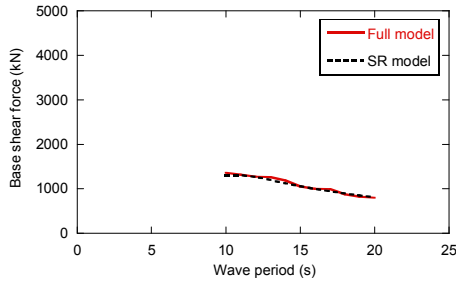
For sway mode, the shear force at node i can be obtained from the tower base response:

$$Q_i^S(t) = \sum_{k=i}^n m_k \ddot{x}_k^S = \sum_{k=i}^n m_k a_S(t) \frac{\sum_{j=1}^n |H_j^S(\omega)| \phi_{i,j}^S \phi_{k,j}^S}{\sum_{j=1}^n |H_j^S(\omega)| \phi_{j,j}^S} \quad (8)$$

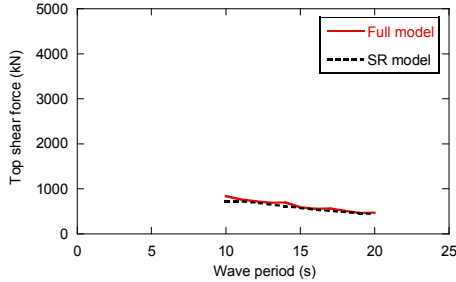
where $a_S(t)$ is the known sway acceleration at tower base. If only the first mode is considered, the shear force becomes

$$Q_i^S(t) = \sum_{k=i}^n m_k a_S(t) \frac{\phi_{k1}^S}{\phi_{i1}^S} = \sum_{k=i}^n m_k a_S(t) \left(1 + \frac{\Delta \phi_{k1}^S}{\phi_{i1}^S} \right) \quad (9)$$

where $\Delta \phi_{k1}^S = \phi_{k1}^S - \phi_{i1}^S$. $\Delta \phi_{k1}^S / \phi_{i1}^S$ is defined as the elastic/solid ratio of mode shape at node k .



(a) Base shear force



(b) Top shear force

Fig. 5 Shear force on wind turbine tower of tension leg system

For rocking mode, the linear acceleration \ddot{x}_k^R at node k can be calculated from the angular acceleration which can be obtained from modal analysis. Like sway effect, the shear force at node i due to rocking motion can also be obtained from the tower base response:

$$Q_i^R(t) = \sum_{k=i}^n m_k \ddot{x}_k^R = \sum_{k=i}^n m_k a_R(t) \frac{\sum_{j=1}^n |H_j^R(\omega)| \phi_{i,j}^R \phi_{k,j}^R}{\sum_{j=1}^n |H_j^R(\omega)| \phi_{j,j}^R} (h_{r+1} - h_r) \quad (10)$$

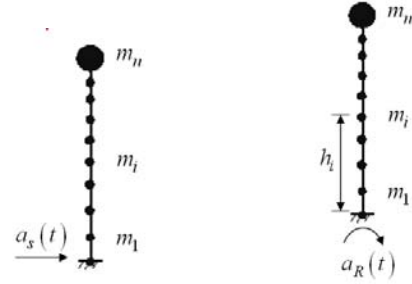
where $a_R(t)$ is the known rocking acceleration at tower base. If only the first mode is considered, the shear force becomes

$$Q_i^R(t) = \sum_{k=i}^n m_k a_R(t) \left(h_k + \sum_{r=1}^{k-1} \frac{\Delta \phi_{r1}^R}{\phi_{i1}^R} (h_{r+1} - h_r) \right) \quad (11)$$

where $\Delta \phi_{r1}^R = \phi_{r1}^R - \phi_{i1}^R$. $\Delta \phi_{r1}^R / \phi_{i1}^R$ is defined as the elastic/solid ratio of mode shape at node r .

Shear Force of Fixed-foundation Model

The sway and rocking acceleration at tower base from the FEM simulation of floating wind turbine system will be used to a fixed-foundation wind turbine in each corresponding direction.



(a) With sway acceleration (b) With rocking acceleration

Fig. 6 Fixed-foundation model

Fig. 6 (a) shows the fixed-foundation model with sway acceleration. The modal equation of motion is expressed as:

$$M_j \ddot{f}_j^s(t) + C_j \dot{f}_j^s(t) + M_j \omega_j^2 f_j^s(t) = -a_S(t) \sum_{k=1}^n m_k \phi_{kj} \quad (12)$$

where ϕ_{kj} is the normalized mode shape of the j th mode for fixed-foundation model. The modal displacement can be shown as:

$$f_j^s(t) = \frac{-a_S(t) \sum_{k=1}^n m_k \phi_{kj}}{M_j \omega_j^2} \cdot \frac{1}{\sqrt{(1 - \beta_j^2)^2 + 4\xi_j^2 \beta_j^2}} \quad (13)$$

The structural relative acceleration at node k for the j th mode is:

$$\ddot{x}_{kj}^s(t) = \ddot{f}_j^s(t) \cdot \phi_{kj} = -\omega^2 f_j^s(t) \cdot \phi_{kj} = \gamma_j^s \phi_{kj} A_j(\beta_j, \xi_j) a_S(t) \quad (14)$$

$$\text{where } \gamma_j^s = \frac{\sum_{k=1}^n m_k \phi_{kj}}{\sum_{k=1}^n m_k \phi_{kj}^2}$$

$$A_j(\beta_j, \xi_j) = \frac{\beta_j^2}{\sqrt{(1 - \beta_j^2)^2 + 4\xi_j^2 \beta_j^2}}$$

γ_j^s is the well-known participation factor, $\beta_j = \omega / \omega_j$ is the ratio between external wave frequency and structural natural frequency, and ξ_j is the structural damping ratio, and referring to Ishihara (2010), $\xi_j = 0.5\%$ is used here. The shear force at node i of tower can be calculated as:

$$Q_{i,s}^F(t) = \sum_{k=i}^n m_k \left(\sum_{j=1}^n \gamma_j^s \phi_{kj} A_j(\beta_j, \xi_j) a_S(t) + a_S(t) \right) \quad (15)$$

If only the first mode is considered, the shear force becomes

$$Q_{i,s}^F(t) = \sum_{k=i}^n m_k a_S(t) \left(1 + \gamma_1^s \phi_{k1} A_1(\beta_1, \xi_1) \right) \quad (16)$$

Fig. 6 (b) shows the fixed-foundation model with rocking acceleration.

The shear force at node i can be obtained from the modal analysis as well, as shown in Eq. 17.

$$Q_{i,r}^F(t) = \sum_{k=i}^n m_k \sum_{j=1}^n \gamma_j^r \phi_{kj} A_j(\beta_j, \xi_j) a_R(t) + \sum_{k=i}^n m_k a_R(t) h_k \quad (17)$$

where
$$\gamma_j^r = \frac{\sum_{k=1}^n m_k \phi_{kj} h_k}{\sum_{k=1}^n m_k \phi_{kj}^2}$$

γ_j^r is the participation factor for the rocking direction. If only the first mode is considered, the shear force becomes

$$Q_{i,r}^F(t) = \sum_{k=i}^n m_k a_R(t) (h_k + \gamma_1^r \phi_{k1} A_1(\beta_1, \xi_1)) \quad (18)$$

Comparison of Shear Force

Taking the sway direction as example, from Eqs. 9 and 16 it is found that both the shear forces of the SR model and fixed-foundation model consist of solid part and elastic part, as shown in Fig.7 (a). The solid parts are totally same, but the elastic parts are different. Fig. 7 (b) compares the elastic parts of shear force from the two models. In the real situation, since $\Delta\phi_{n1}^S / \phi_{11}^S$ of floating wind turbine system is usually less than 15%, the elastic/solid ratio of shear force will be less than 0.08, and doesn't change with wave period; while for fixed-foundation model, the elastic part of shear force is the function of T/T_1 , the ratio between external wave period and structural natural period. It is noticed that when $T/T_1 > 4$, the fixed-foundation model underestimates the shear force, while when $T/T_1 < 4$, it may give significant overestimation, which can be larger than 15%. Especially when T/T_1 becomes close to 1, the resonance would happen, so in this case this model is not reasonable at all. Therefore, the fixed-foundation model can not be used as the calculating model for floating wind turbine system.

PREDICTION OF WAVE-INDUCED LOAD

Based on the above discussion, the shear force on tower can be predicted using sway-rocking model with modal analysis. Both regular wave and irregular wave are considered here for tension leg mooring system and catenary mooring system, respectively.

Wave Conditions

The linear Airy wave is used to derive the shear force with modal analysis, since this kind of regular wave has single wave period and is easier to explain the effect of external frequency on the structural response. The extreme wave height $H_{extreme} = 20m$ and wave periods varying from 10s - 20s at intervals of 1s are used in regular wave case.

In the real situation, the irregular wave which is represented by the significant wave height H_s and the spectral peak period T_p should be used. In this study the extreme 3-hour sea state with a 50-year recurrence period is considered. In the short term, i.e. over a 3-hour or 6-hour period, stationary wave conditions with constant H_s and constant T_p are assumed to prevail (IEC-61400-3, 2008). Thus, significant wave height 10.75m and peak wave period varying from 10~20 sec at an interval of 1s are used in this research. The time history of wave elevation is generated using JONSWAP spectrum. The peak

factor of wave is determined as 3.3; the shape factor is 0.07 for $\omega \leq 2\pi/T_p$ and 0.09 for $\omega > 2\pi/T_p$ according to Chakrabarti (1987). Here, ω is the angular frequency of wave.

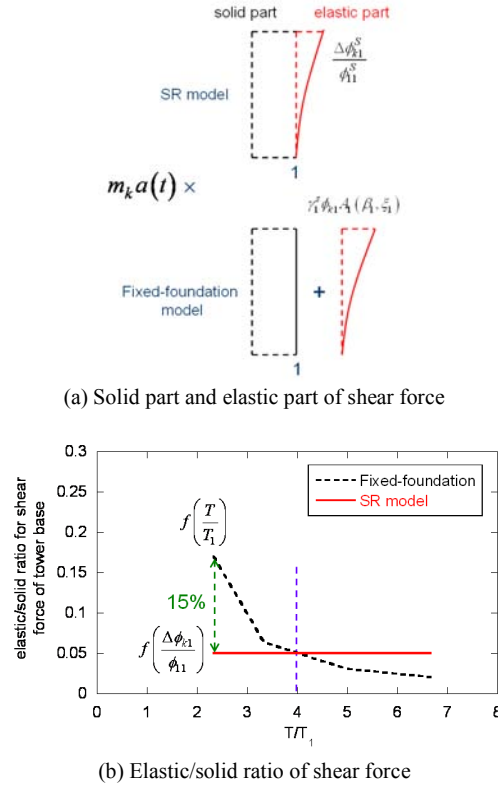


Fig. 7 Comparison of elastic parts of shear force

Tower Loading under Regular Wave

For regular wave case, in Eqs. 9 and 11, $a_S(t)$ and $a_R(t)$ can be replaced by their respective amplitude to calculate the amplitudes of shear force on tower due to sway motion and rocking motion accordingly.

For tension leg system, the rocking motion has no contribution to the shear force, and the sway motion can determine the total load. While for catenary system, both sway motion and rocking motion have significant effect on tower loading of catenary system. Hence, the influence of the two motions should be combined together. From the FEM simulation, it is recognized that the maximum response of sway and rocking don't occur concurrently, but a certain correlation exists between them. Referring to the seismic loads specified in AIJ (2004), complete quadratic combination (CQC) is used here for the combination. It is noted that the correlation between sway and rocking modes doesn't change with the external excitation, i.e., wave force, and it only depends on the damping and natural frequency of the system.

Referring to AIJ (2004), there is another method for sway-rocking combination: square root of sum of squares (SRSS), which takes the correlation factor as 0. Fig.8 indicates that SRSS rule underestimates the shear force. As the sway and rocking modes have closer eigenvalues, CQC rule can give much better results.

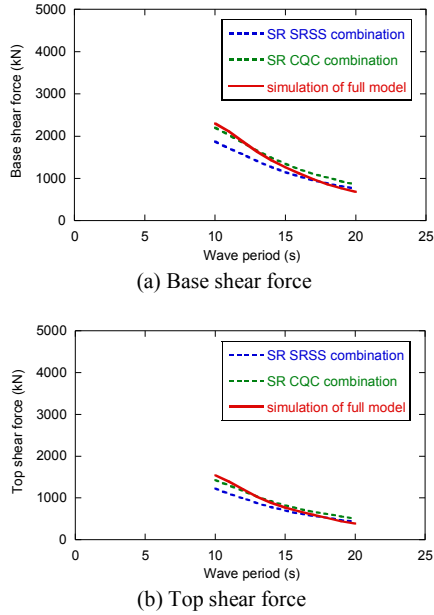


Fig. 8 Comparison of shear force from CQC and SRSS

Tower Loading under Irregular Wave

For irregular wave, the tower loading is a random process. Hence, the standard deviation and peak factor is considered and their product is used to calculate the maximum load in the equivalent static method. Based on the same idea as regular wave case, in Eqs. 9 and 11, $a_s(t)$ and $a_r(t)$ can be replaced by their respective standard deviation to calculate the shear force standard deviation due to sway motion and rocking motion accordingly. For tension leg system, the sway motion can determine the total standard deviation; while for catenary system the standard deviation of shear force can also be considered as the combination of sway effect and rocking effect using CQC rule.

Before proposing the formulae of peak factor, power spectrum density of tower base shear force is investigated. Fig.9 shows the comparison of power spectrum density of tower base shear force for the wave periods: 10s, 15s and 20s. It is noted that the dynamic tower loading consists of three parts. Taking the 10s case of tension leg system in Fig.9 (a) as example, the range around the first peak is the background motion part, which has the same peak frequency as the wave $n_p = 0.1$, corresponding to the wave peak period 10s; The range around the second peak is due to the peak acceleration of the floater sway motion with the peak frequency $n_s = 0.167$, corresponding to the peak period of sway acceleration 6s; The range around the third peak is the resonant part due to the tower vibration with the peak frequency $n_1 = 0.289$, corresponding to the natural period of tower 3.5s. The second and third peaks result in the non-Gaussian characteristics of the shear force. From the comparison with 15s case and 20s case, it is found that when the wave period becomes longer, the two peaks will be reduced since the frequency difference from the wave becomes larger and external exciting effect becomes weaker, which means the non-Gaussianity will decrease when wave period increases. This feature is just the reason why the skewness α_3 of tower base shear force for tension leg system in Fig.10 is significant for 10s-15s, and can be neglected after 16s. This skewness can be obtained from the simulation of tension leg floating

wind turbine system. Therefore, the tower loading is considered as a non-Gaussian process for tension leg system. Based on the model of Kareem (1998), the non-Gaussian peak factor for the tower shear force under irregular wave is proposed as Eq.19.

For catenary system as shown in Fig.9 (b), the second and third peaks are negligibly small compared to the background motion part, since the floater sway and rocking modes are much more dominant, and the tower resonance is only slightly excited. In 15s case and 20s case, the two peaks will not exist, and only the background motion part is left. As a result a Gaussian process can be assumed for the tower base shear force of catenary system. This feature is also the reason why the skewness for catenary system in Fig.10 is close to zero for all wave periods. With $\alpha_3 = 0$, the non-Gaussian peak factor of Eq.19 is reduced to the Gaussian form.

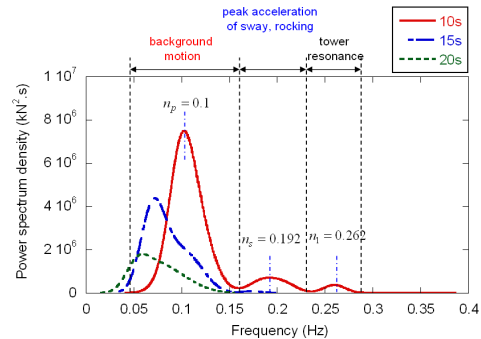
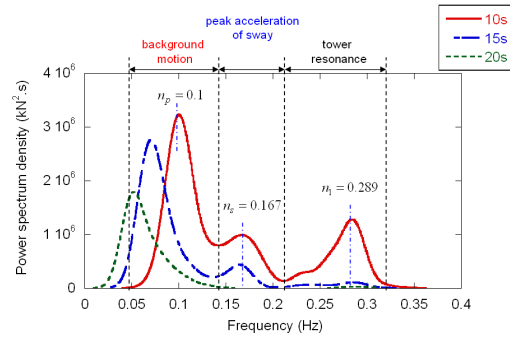


Fig. 9 Comparison of power spectrum density of tower base shear force

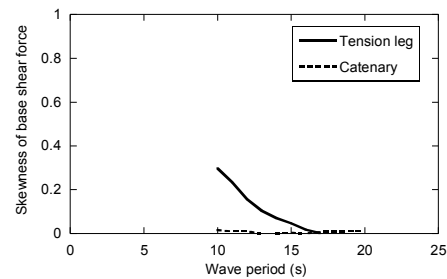


Fig.10 Comparison of skewness of tower base shear force between tension leg system and catenary system

$$g = \frac{1}{\sqrt{1 + \frac{\alpha_3^2}{18}}} \left\{ \left(\sqrt{2 \ln(\nu_0' T)} + \frac{0.5772}{\sqrt{2 \ln(\nu_0' T)}} \right) + \frac{\alpha_3}{6} (2 \ln(\nu_0' T) - 1) \right\} \quad (19)$$

where ν_0' and ν_0 are the zero up-crossing frequency of tower base shear force for non-Gaussian process and Gaussian process, respectively, and $\nu_0' = \nu_0 / \sqrt{(1 + \alpha_3^2/18)(1 + \alpha_3^2/9)}$.

From Fig.11, it is indicated that the non-Gaussian peak factor is necessary for tension leg system and it decreases with the wave period since the skewness and zero up-crossing frequency have the same tendency. The Gaussian peak factor is enough for catenary system and it doesn't change much with wave period. The larger difference between non-Gaussian and Gaussian peak factors happens in the shorter wave periods of 10s~15s.

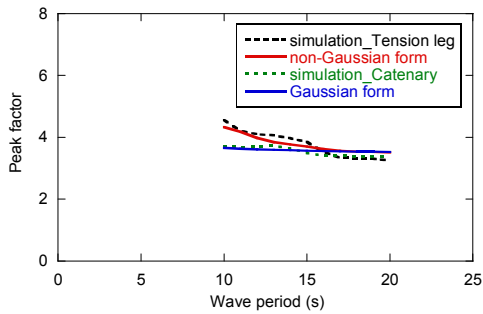


Fig. 11 Comparison of peak factors between tension leg system and catenary system

CONCLUSIONS

An equivalent SR model is proposed to consider the influence of floater surge and pitch motions on the tower loading of floating wind turbine systems. A theoretical comparison of shear force by modal analysis is performed between SR model and fixed-foundation model. Finally, the evaluation formulae of tower loading is proposed theoretically based on the equivalent static method. The conclusions are summarized as follows:

- (1) Through the theoretical comparison between SR model and fixed-foundation model, it is found that in short wave period, the fixed-foundation model may give significant overestimation, which can be larger than 15%; while in long wave period, it underestimates the tower loading.
- (2) The evaluation formulae of tower loading due to sway as well as rocking motion of floater are investigated separately by locking

the other mode with modal analysis. Their combination is calculated with CQC rule. The correlation between them only depends on the damping and natural frequency of the system.

- (3) Under irregular wave, for tension leg system, a non-Gaussian peak factor is necessary due to the tower resonance mainly. The non-Gaussianity will decrease with wave period, since the external exciting effect becomes weaker. For catenary system, the shear force history can be regarded as a Gaussian process. The effect from tower resonance is negligibly small compared to the background motion part, since the floater sway and rocking modes are much more dominant.

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