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A peak factor for non-Gaussian response analysis of wind turbine tower

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Abstract

Equivalent static wind load evaluation formulas considering the dynamic effects based on peak factor were proposed to estimate the design wind load on the wind turbine tower in complex terrain. The non-linear part of wind pressure was considered to estimate the mean wind loads. The peak factor based on a non-Gaussian assumption was derived to estimate the non-linearity of wind load, especially in the high turbulence intensity. The formula of the peak factor is simplified to a function of the third order moment (skewness) considering the spatial correlation of wind velocity, the resonance response and the background response. The proposed methods showed favorable agreements with dynamic wind response analysis by FEM.

Keywords: Wind turbine tower; Non-Gaussian response; Peak factor; Skewness; Spatial correlation; Resonance

1. Introduction

Wind load on wind turbine is usually evaluated either by finite element model (FEM) or by equivalent static method. While FEM simulation is commonly used in turbine design, equivalent static method is used widely in design of lower and other support structures. Equivalent static method is adopted in many design codes (recommendations for loads on

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buildings, Architectural Institute of Japan, 1993; Danish standard DS472, The Danish Society of Engineers and The Federation of Engineers, 1992). This method uses a coefficient called the peak factor proposed by Davenport (1964) to account for fluctuating wind load.

In formulas of mean wind load, standard deviation and peak factor of fluctuating wind load proposed in codes, the non-linear part of wind pressure is neglected. Therefore, if a structure is under high turbulence intensity, mean wind load and peak factor may be underestimated, since contribution of the non-linear part of wind pressure is large and the response is non-Gaussian. Kareem and Zhao (1994) proposed a formula for peak factor, which can be applied to non-Gaussian process and confirmed its validity in the case of a single degree of freedom system through numerical simulation. Ishikawa (2004), meanwhile, pointed out that Kareem's formula gives conservative results, especially when spatial correlation of wind velocity is considered. Using the moment-based Hermite transformation method and the definition of peak factor proposed by Nishijima et al. (2002), Ishikawa (2004) proposed a formula for peak factor which considers both non-Gaussianity and spatial correlation of wind load on transmission line. However, this formula neglects resonance response due to high damping ratio of the transmission line.

A wind turbine is characterized by a low structural damping and a heavy head, which results in significant resonant response. Besides, wind turbines exposed to high wind turbulence in areas with complex terrain like Japan can exhibit strong non-Gaussian responses; and with the rapid increase of wind turbine size, considering spatial correlation is essential.

This study proposes a formula of the maximum wind load on wind turbines in complex terrain. The mean wind load, which considers the non-linear part of wind pressure, is derived. The non-Gaussian peak factor, which takes into account both the spatial correlation of wind velocity and resonance response, is proposed. The formula is verified by FEM using the wind turbine investigated by Ishihara et al. (2005).

2. Wind turbine model

In this study the model of an elastic tower and a rigid rotor, shown in Fig. 1b, is used to implement the theoretical formula of mean, standard deviation and the peak factor of wind load on a tower base. Parameters of the formula of the peak factor are determined by the results of FEM simulation. Since in wind load of wind turbine tower the effect of the first mode is dominant, only the first mode is considered. The effect of higher modes is negligibly small, because of the low power in high frequency region of the spectrum of wind load. However, it should be noted here that in the case of seismic load, where the power spectrum is high in high frequency regions, the effect of higher modes is not negligible.

Wind velocity and turbulence intensity at the hub of the wind turbine are used as representative for that of the whole rotor. Wind load on the rotor is calculated and transferred to the tower as shear force and bending moment at top of the tower.

3. Equivalent static method for wind turbine

To illustrate this method, let us start with a simple model of wind turbine response

$$M\ddot{x} + C\dot{x} + Kx = F_{\mathrm{tot}}, \tag{1}$$

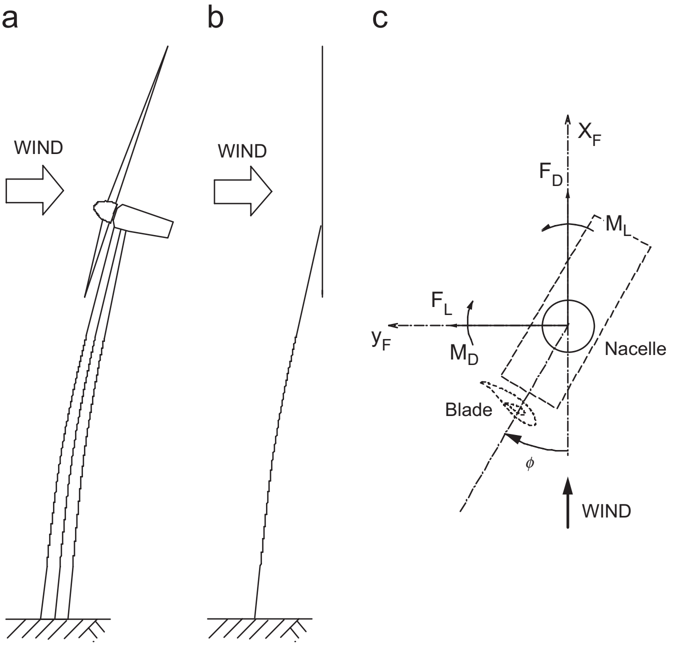

Fig. 1. Wind turbine model and wind direction definition. (a) Wind turbine; (b) Simplified model; (c) Wind direction and wind load.

where M is the mass matrix, C is the damping matrix and K is the stiffness matrix; and

$$F_{\text{tot}} = \tfrac{1}{2}\rho C_{\text{f}} S(U+u)^2 = \tfrac{1}{2}\rho C_{\text{f}} S(U^2 + 2Uu + u^2), \tag{2}$$

where ρ is the density of air, C_{f} is the aerodynamic force coefficient, S is the considered area, U is the mean wind velocity and u is the fluctuating wind velocity.

3.1. Mean wind load

From (2) the mean wind force and bending moment can be derived:

$$\overline{F_{\text{tot}}} = \tfrac{1}{2}\rho C_{\text{f}} S(U^2 + \sigma_u^2) = \tfrac{1}{2}\rho C_{\text{f}} S U^2 (1 + I_u^2), \tag{3}$$

$$M = \int_R \tfrac{1}{2}\rho C_{\text{f}}(x) U^2 (1 + I_u^2) c(x) x \, \mathrm{d}x, \tag{4}$$

where I_u is the turbulence intensity, $c(x)$ is the characteristic size of the element at position x and R denotes all over wind turbine.

A study by Kareem and Zhou (2003) proved that the bending moment-based peak factor can yield more reliable results than displacement-based peak factor, because the mean value of displacement may be zero. Therefore, in this study, the bending moment-based peak factor is adopted. This means the term wind load should be interpreted as a bending moment.

3.2. Standard deviation

Standard deviation of fluctuating wind load consists of a background part σ_{MB} and a resonant part σ_{M1}:

$$\sigma_M = \sqrt{\sigma_{MB}^2 + \sigma_{M1}^2}. \tag{5}$$

From (2) and (3) fluctuating wind force can be calculated:

$$F_t = \tfrac{1}{2}\rho C_f S(U+u)^2 - \tfrac{1}{2}\rho C_f S U^2(1+I_u^2) = \rho C_f SUu + \tfrac{1}{2}\rho C_f S u^2 - \tfrac{1}{2}\rho C_f S U^2 I_u^2. \tag{6}$$

Therefore, the background standard deviation of wind load can be calculated by dividing the bending moment caused by F_t to mean bending moment M.

$$\frac{\sigma_{MB}}{M} = \frac{2I_u}{1+I_u^2}\sqrt{K_{SMB} + K'_{SMB}}, \tag{7}$$

$$K_{SMB} = \frac{\int_R \int_R (\frac{1}{2}\rho C_f U_1^2)(\frac{1}{2}\rho C_f U_2^2)\rho_{12} c(x_1)c(x_2) l_1 l_2 \, dx_1 \, dx_2}{\left(\int_R \frac{1}{2}\rho C_f U^2 c(x) l \, dx\right)^2}, \tag{8}$$

$$K'_{SMB} = \frac{1}{2} I_u^2 \frac{\int_R \int_R (\frac{1}{2}\rho C_f U_1^2)(\frac{1}{2}\rho C_f U_2^2)\rho_{12}^2 \times c(x_1)c(x_2) l_1 l_2 \, dx_1 \, dx_2}{\left(\int_R (\frac{1}{2}\rho C_f U^2) c(x) l \, dx\right)^2}, \tag{9}$$

where ρ_{12} is the cross correlation of wind velocity at x_1 and x_2, l, l_1, l_2 are the bending lever arms of elements at x, x_1, x_2 about the tower base, respectively.

The bending lever arm l is the distance from the considering point to tower base if that point is on the tower. If the considering point is on the rotor then l is the distance from top of the tower to the tower base. Calculation of the integrals in (8) and (9) is implemented by a computer program which uses two lists of wind turbine elements to consider all available correlations. The number of lists becomes three and four for three-fold or four-fold integrals.

The resonant part of standard deviation which considers only the first mode of tower can be derived from modal analysis, as follows:

$$\frac{\sigma_{M1}}{M} = \frac{\sqrt{\pi} I_u \lambda_{M1}}{\sqrt{\xi}} \sqrt{R_u(n_1) K_{Sx1}(n_1)}, \tag{10}$$

$$K_{Sx1} = \sqrt{\frac{\int_0^R \int_0^R Co_u^{(x_1,x_2,n)} \mu_1(x_1)\mu_1(x_2) c(x_1)c(x_2) \, dx_1 \, dx_2}{\left(\int_0^R \mu_1(r) c(r) \, dr\right)}}, \tag{11}$$

$$\lambda_{M1} = \frac{\int_0^R m(r)\mu_1(r) r \, dr}{m_1 \int_0^R c(r) r \, dr} \int_0^R c(r)\mu_1(r) \, dr, \tag{12}$$

where ξ is the structural damping ratio, R_u is the normalized power spectrum of wind, n_1 is the first modal frequency of the structure. $Co_u^{(x_1,x_2,n)}$ is the normalized co-spectrum of wind velocity and μ_1 is the first mode shape.

3.3. Peak factor

A widely adopted model in codes is the peak factor model proposed by Davenport (1964). Assuming that wind response of structure is a Gaussian process, the formula is given as

$$g = \sqrt{2\ \ln(vT)} + \frac{0.5772}{\sqrt{2\ \ln(vT)}}, \tag{13}$$

$$v = \sqrt{\frac{\int_0^\infty n^2 S_M(n)\,\mathrm{d}n}{\int_0^\infty S_M(n)\,\mathrm{d}n}} = \sqrt{\frac{n_0^2\sigma_{MB}^2 + n_1^2\sigma_{M1}^2}{\sigma_{MB}^2 + \sigma_{M1}^2}}, \quad n_0 = \sqrt{\frac{\int_0^\infty n^2 S_u(n)\,\mathrm{d}n}{\int_0^\infty S_u(n)\,\mathrm{d}n}}, \tag{14}$$

where v is the zero up-crossing number in a unit of time of a Gaussian process, T is the estimated time interval (normally $T = 600\,\mathrm{s}$), S_M is the power spectrum of wind load, S_u is the power spectrum of wind velocity, and n is the frequency variable.

4. Peak factor model

In order to take the non-linear component of wind load into account, Ishikawa (2004) derived a formula for the peak factor using the definition of Nishijima et al. (2002) in which the peak factor of a process is the value that the process up-crosses once on average in a certain time T:

$$g = \kappa\left\{\sqrt{2\ \ln\ v'_yT} + h_3(2\ \ln\ v'_yT - 1) + h_4[(2\ \ln\ v'_yT)^{3/2} - 3\sqrt{2\ \ln\ v'_yT}]\right\}, \tag{15}$$

$$\kappa = \frac{1}{\sqrt{1 + 2h_3^2 + 6h_4^2}}, \quad v'_Y = \frac{1}{\sqrt{1 + 4h_3^2 + 18h_4^2}} v_y,$$
$$h_3 = \frac{\alpha_3}{4 + 2\sqrt{1 + (3(\alpha_4 - 3))/2}}, \quad h_4 = \frac{\sqrt{1 + (3(\alpha_4 - 3))/2} - 1}{18}, \tag{16}$$

where α_3, α_4 are the third, forth order moments of wind load, respectively, v'_y is the zero up-crossing number in T of the non-Gaussian process Y' and v_y is the zero up-crossing number in T of a Gaussian process Y which can be calculated by (14).

From formula of α_3 and α_4 derived by Ishikawa (2004), the effect of the forth order part α_4 is neglected since it is negligibly small compared to that of the second and third order from the order analysis of turbulence intensity I_u. α_4 is then assumed to be equal to the value of a Gaussian process (i.e., 3.0) and the expression of peak factor becomes

$$g = \kappa\left\{\sqrt{2\ \ln\ v'_YT} + h_3(2\ \ln\ v'_YT - 1)\right\}, \tag{17}$$

$$h_3 = \frac{\alpha_3}{6}, \quad v'_Y = \frac{1}{\sqrt{1 + (\alpha_3^2/9)}} v_y, \quad \kappa = \frac{1}{\sqrt{1 + (\alpha_3^2/18)}}. \tag{18}$$

In this model, the skewness of fluctuating wind load is necessary to calculate the peak factor g. A model for skewness of wind load on transmission line, proposed by Ishikawa

(2004), is as follows:

$$\alpha_3 = \frac{3I_u a_{r1} + I_u^3 a_{r2}}{(K_{SMB} + K'_{SMB})^{3/2}}, \tag{19}$$

$$a_{r1} = \frac{\int_0^L \int_0^L \int_0^L (\frac{1}{2}\rho C_f U_1^2)(\frac{1}{2}\rho C_f U_2^2)(\frac{1}{2}\rho C_f U_3^2)\rho_{12}\rho_{23} c(x_1)c(x_2)c(x_3)x_1x_2x_3\,\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3}{\left(\int_0^L (\frac{1}{2}\rho C_f U^2)c(x)x\,\mathrm{d}x\right)^3} \tag{20}$$

$$a_{r2} = \frac{\int_0^L \int_0^L \int_0^L (\frac{1}{2}\rho C_f U_1^2)(\frac{1}{2}\rho C_f U_2^2)(\frac{1}{2}\rho C_f U_3^2)\rho_{12}\rho_{23}\rho_{13} c(x_1)c(x_2)c(x_3)x_1x_2x_3\,\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3}{\left(\int_0^L (\frac{1}{2}\rho C_f U^2)c(x)x\,\mathrm{d}x\right)^3}, \tag{21}$$

where L is the length of the transmission line.

It is noted that these formulas do not consider resonance response. Therefore, they cannot be applied directly to wind turbines. In this study, a function of resonance response is introduced into (19). This is a function of the resonance–background ratio R_d of standard deviation denoted by $f(R_d)$. Since I_u^3 and K'_{SMB} are negligibly small compared to I_u and K_{SMB}, respectively, the expression of α_3 in (19) becomes

$$\alpha_3 = f(R_d) \times \frac{3I_u a_{r1}}{(K_{SMB})^{3/2}}, \tag{22}$$

Table 1
Description of the FEM code

Name	Description
Dynamic analysis	Direct numerical integration, the Newmark method
Eigenvalue analysis	Subspace iteration procedure
Element type	Beam element
Formulation	Total Lagrangian formulation
Aerodynamic force	Quasi-steady aerodynamic theory
Damping	Rayleigh damping

Table 2
Main characteristics of the wind turbine studied

Name	Description
Rated power	400 kW
Regulation	Stall
Number of blades	3
Rotor diameter	31 m
Hub height	36 m

$$R_d = \frac{\sigma_{M1}}{\sigma_{MB}}. \tag{23}$$

The FEM code, developed by Ishihara et al. (2005), is described in Table 1. The main idea is using an aerodynamically and structurally modeled beam element to model wind turbine tower and blades. Wind series at all nodes are generated by a correlation matrix and wind load derived from these series is used in the equation of motion. The FEM program is used to simulate the response of the wind turbine model described in Table 2 and Fig. 2 with

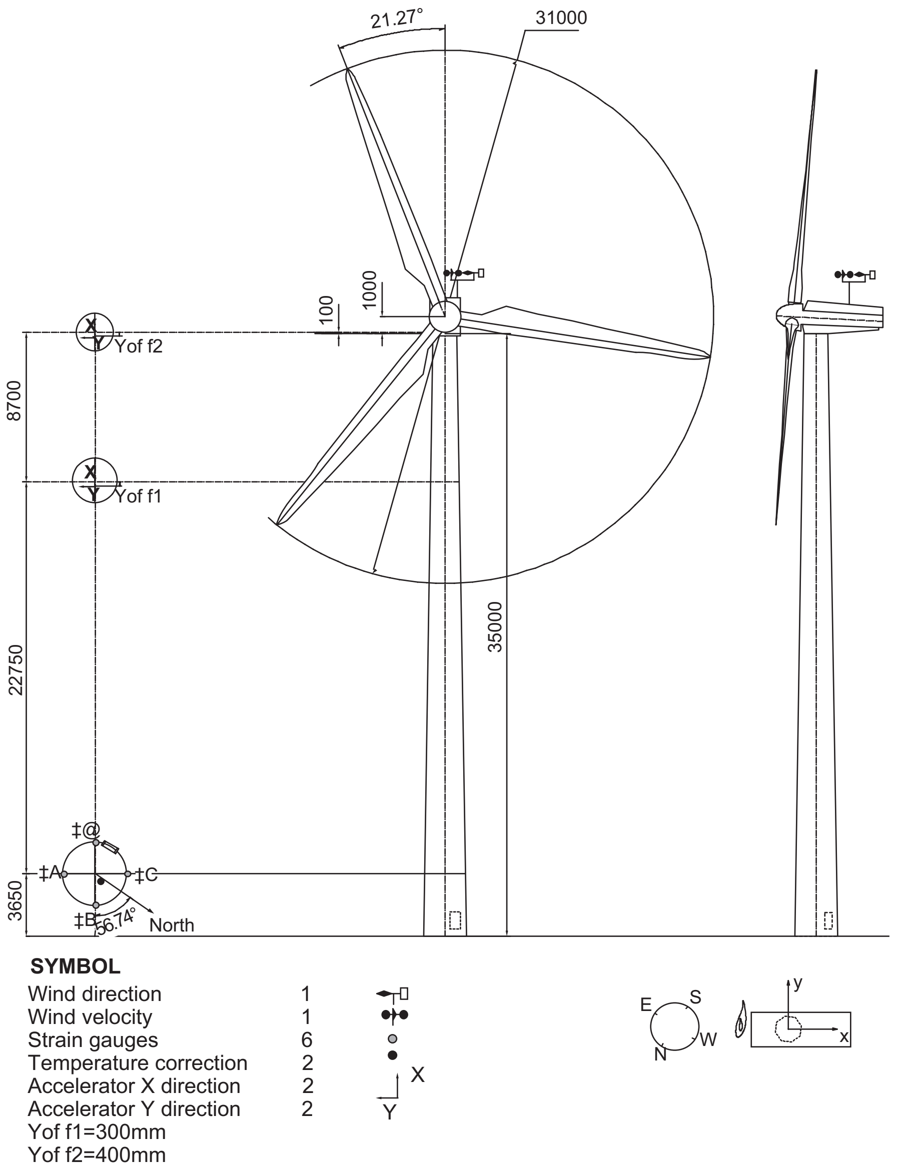

Fig. 2. Configuration of wind turbine.

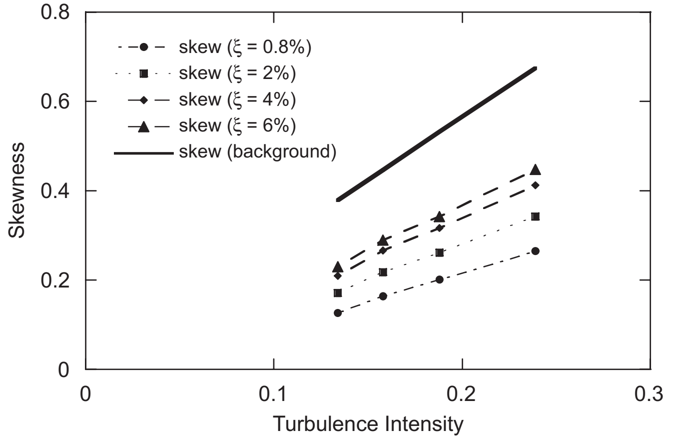

Fig. 3. Relation of skewness, turbulence intensity and damping.

different structural damping. The design wind speed at hub is 50 m/s. The power law for wind shear and turbulence intensity of different terrain categories described in Architectural Institute of Japan's (1993) recommendations for loads on buildings is adopted. Results in Fig. 3 show that skewness and turbulence intensity have a linear relationship, which confirms the validity of formula (22). It is also noticed that skewness increases when damping ratio increases. Since the damping ratio of wind turbine ξ varies in a narrow range from 0.005 to 0.01, it can be assumed that the skewness α_3 and the damping ratio ξ have a linear relation. Therefore, $f(R_d)$ is supposed to be proportional to the damping ratio ξ (i.e., proportional to R_d^{-2}), since the damping ratio ξ is proportional to R_d^{-2}. It is also noticed that $f(R_d)$ should become 1 if there is no resonance (i.e., when $R_d = 0$). Therefore the following form of $f(R_d)$ is proposed and α can be derived:

$$f(R_d) = \frac{1}{\alpha R_d^2 + 1}, \tag{24}$$

$$\alpha = \frac{1}{R_d^2}\left[\frac{3I_u a_{r1}}{(K_{SMB})^{3/2}}\frac{1}{\alpha_3} - 1\right]. \tag{25}$$

In order to determine α, FEM wind response simulations of wind turbine of different R_d (i.e., different damping ratio ξ) were carried out to calculate skewness α_3. Other parameters are calculated from theoretical formula. Finally, α is calculated by formula (25). From the result in Fig. 4 the conservative value $\alpha = 1.3$ is proposed. The formula of skewness α_3 becomes

$$\alpha_3 = \frac{1}{1.3R_d^2 + 1} \times \frac{3I_u a_{r1}}{(K_{SMB})^{3/2}}. \tag{26}$$

In this model of skewness, the peak factor decreases when skewness decreases. Since skewness decreases when R_d increases, the peak factor decreases when R_d increases. Because R_d increases when the resonant load increases, the peak factor decreases when the

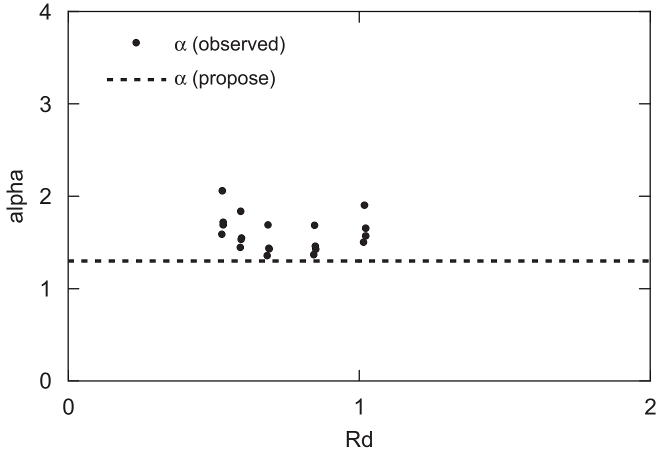

Fig. 4. Resonant Coefficient α.

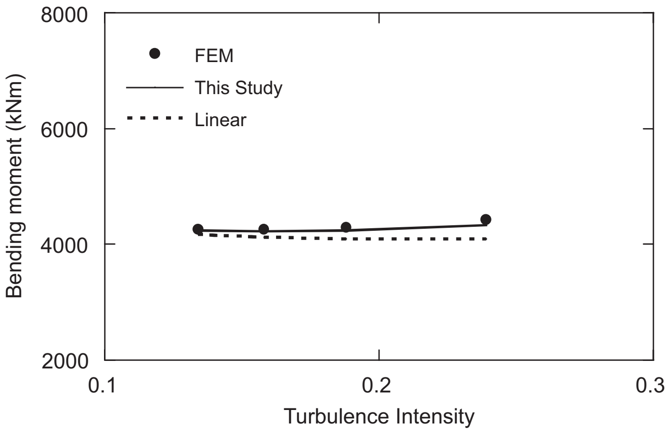

Fig. 5. Comparison of mean load.

resonant load increases. This model agrees well with the study by Kitada et al. (1991) which states that the peak factor decreases when the correlation of peaks increases, because an increase of the resonant load means that peaks occur in a certain manner and the correlation of peaks increases.

5. Verification of proposed model

The proposed formulas are used to calculate design wind load on the wind turbine tower described in Table 2 and Fig. 2 with the same wind conditions described in Section 4. Since in codes, the largest wind load is considered to be drag force when wind flows from in front of wind turbine (i.e., the inflow angle is zero), this load case is investigated. Figs. 5–8 are examples of how these results strongly correlate with the FEM simulation in both low and high turbulence intensity, which means the formulas can be used to estimate wind load on wind turbine towers in complex terrain.

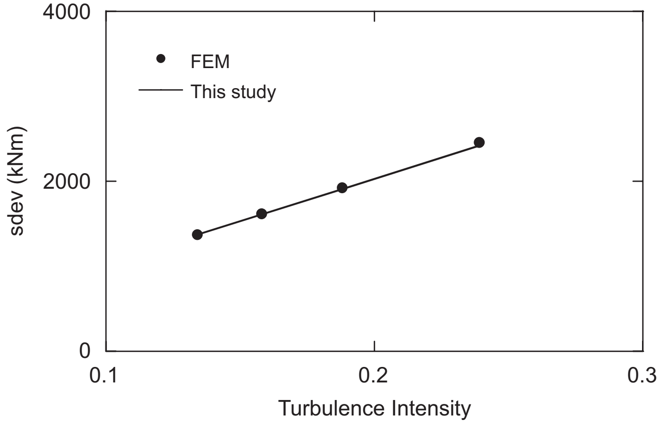

Fig. 6. Comparison of standard deviation.

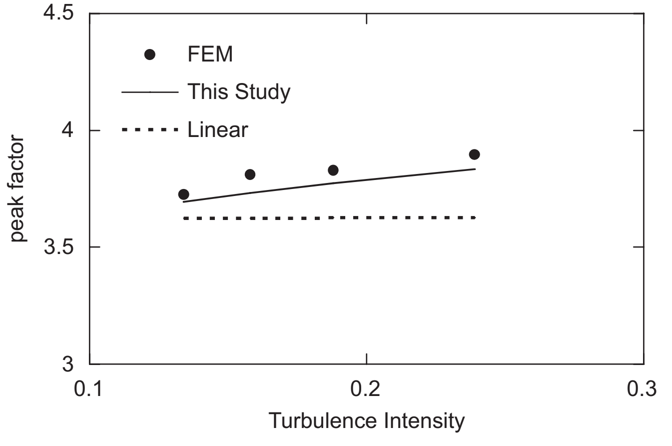

Fig. 7. Comparison of peak factor.

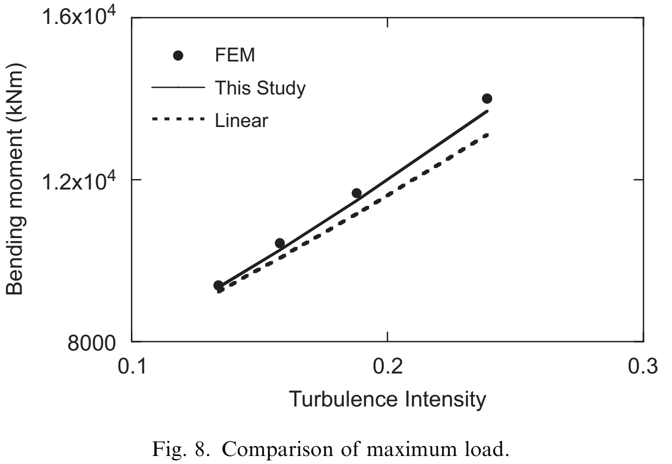

Fig. 8. Comparison of maximum load.

6. Concluding remarks

In this study, an equivalent static method to evaluate wind load on wind turbines has been studied. The followings were obtained:

(1) A formula of mean wind load, which considers the non-linear part of wind pressure, was proposed to evaluate wind load in region of high turbulence intensity.
(2) A formula of peak factor was proposed to consider a non-Gaussian response of a wind turbine tower by introducing skewness. Proposed skewness formula, which considers the spatial correlation of wind velocity, turbulence intensity and the resonance–background ratio of wind load, consists of a theoretical background part and an empirical turbulent part.
(3) The formulas have been verified using FEM simulation of a stall-regulated wind turbine. Especially in regions of high turbulence, the calculated load's error is limited to less than five percent.

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