The Second International Symposium on
Wind and Structures
Busan, Korea, August 21-23, 2002

Prediction of Flutter Characteristics of Rectangular Cross-Sections by k-ε model

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ABSTRACT

In this paper, flutter analysis of rectangular cross-sections based on k-ε model is discussed. Recently, some successful application of the computational fluid dynamics to the simulation of flutter has been reported. It is noteworthy to mention here that almost all of the aeroelastic application has been performed so far by means of two-dimensional analysis. These good performances of 2D analysis is due to the turbulence viscosity which acts as the mimic spanwise momentum diffusion. However, two-dimensional analysis is essentially an approximation and thus it is necessary to examine physical consistency of the obtained results. In this paper, the authors have followed Matsumoto(1994, 2001), who have investigated flutter characteristics of generic rectangular cross-sections, by means of k-ε model and physical consistency of the obtained numerical result is examined. As a result, good agreement was obtained for B/D=5 and 10 cross-sections, however, for B/D=20 cross-section, flutter analysis indicated a conservative prediction in flutter speed.

INTRODUCTION

In this paper, flutter analysis of rectangular cross-sections based on k-ε model is addressed. Recently, some successful application of the computational fluid dynamics to the simulation of flutter has been reported. For example Larsen and Walther(1998) has reported the applicability of the discrete vortex-method in some generic configuration at first and then extended it to some practical bridge cross-sections. The authors has examined k-ε model on the prediction of vortex-

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induced vibration and torsional flutter of generic rectangular cross-sections(Shimada and Ishihara 1999, 2000). It is noteworthy to mention here that almost all of the aeroelastic application has been performed so far by means of two-dimensional analysis. Since in high Reynolds number region, however, three-dimensionality of the flow can not be neglected, these good performances of 2D analysis is due to the turbulence viscosity which is produced from implemented turbulence model or numerical dissipation and acts as the mimic spanwise momentum diffusion. This feature is extraordinarily advantageous on solving the problems involved with the aeroelastic vibration, which require a lot of computational time. However, two-dimensional analysis is essentially an approximation and thus it is necessary to examine physical consistency of the obtained results.

By the way, the characteristics of the aeroelastic vibration such as flutter are characterized by separation and reattachment of the flow. Matsumoto et al.(1994) followed the method proposed by Scanlan and Tomko(1971) and have investigated behavior and mechanism of the unsteady aerodynamic forces in detail using a series of rectangular cross-sections whose B/D ratio(B :chordwise length and D : depth of the cross-section) ranging from 5 to 20. As a result, they showed that torsional flutter and coupled flutter are nothing but a phenomenon which is induced by the continuous change of each component of the unsteady aerodynamic force according to B/D ratio. Furthermore, they also made a detail investigation of the contribution of each component on the flutter characteristics, such as frequency and damping by means of their original "Step-by-step" complex eigenvalue analysis.

In this paper, the same approach is followed by the computational fluid dynamics based on k-ε model and physical consistency of the obtained numerical result with the previous experimental results is examined. Based on this result the application of the k-ε model to the prediction of the flutter is discussed.

NUMERICAL METHOD

The Reynolds-averaged incompressible Navier-Stokes equation is expressed as follows:

$$\frac{DU_i}{Dt} = -\frac{\partial}{\partial x_i}\left(P + \frac{2}{3}k\right) + \frac{\partial}{\partial x_j}\left[(v + v_t)\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\right] \tag{1}$$

where, v_t is the eddy viscosity coefficient and is given as $v_t = C_\mu k^2 / \varepsilon$. Turbulent kinetic energy k and its dissipation rate ε are obtained by the following transport equations:

$$\frac{Dk}{Dt} = \frac{\partial}{\partial x_j}\left[\left(v + \frac{v_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + P_k - \varepsilon \tag{2}$$

$$\frac{D\varepsilon}{Dt} = \frac{\partial}{\partial x_j}\left[\left(v + \frac{v_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] + \left(C_{\varepsilon_1} P_k - C_{\varepsilon_2}\varepsilon\right)\frac{\varepsilon}{k} \tag{3}$$

The empirical parameters in the equation are all of which are identical to those used in the conventional standard k-ε model. P_k is the production term of turbulent kinetic energy. In the present analysis a model proposed by Kato & Launder (1993) in which the production term is modified based on the assumption of flow irrotationality is employed in order to prevent the excessive production of turbulent kinetic energy near the leading edge:

$$P_k = C_\mu \frac{k^2}{\varepsilon} \sqrt{\frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)^2} \sqrt{\frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i}\right)^2} \tag{4}$$

The present method employs two-layer model(Norris & Reynolds 1975). That is, the k equation is solved by assigning k=0 on the solid boundary. Instead of solving ε equation, the ε near the wall is determined by the turbulent kinetic energy k using a length scale l_ε. Eddy viscosity in the region in which ε equation is not solved is calculated using the turbulent kinetic energy k and a length scale l_μ same as ε,

$$\varepsilon = \frac{k^{3/2}}{l_\varepsilon}, \qquad \nu_t = C_\mu k^{1/2} l_\mu \tag{5}$$

The length scale l_ε and l_μ are proportional to turbulent eddy $l(=\kappa y)$ scale and are determined using following relations,

$$l_\mu = C_l y\left[1 - \exp\left(-\frac{R_{e_y}}{A_\mu}\right)\right], \qquad l_\varepsilon = \frac{C_l y}{1 + 5.3 / R_{e_y}} \tag{6}$$

where the constants are given as $C_l = \kappa C_\mu^{-3/4}$ and $A_\mu = 50.5$. In two-layer model since l_ε and l_μ are functions of the turbulent Reynolds number Re_y, the effects of the Reynolds number on flow around bodies with rectangular cross-sections can be evaluated. In the present calculation, the two-layer model is applied to the region only within three meshes away from the solid boundary. The above set of equation is transformed by using a generalized coordinate system and is then solved by the finite difference method. The convective term in the velocity transport equation is discretized by a third-order upwind difference scheme, and the convective terms in the k and ε transport equations are discretized by a first-order upwind difference scheme to stabilize the numerical instability at high Reynolds number arising from the nonlinear effect of the convective term. The pressure is obtained by solving the Poisson equation which is given as,

$$\nabla^2 P = -\frac{\partial}{\partial x_i}\left(U_j \frac{\partial U_i}{\partial x_j}\right) + \frac{\partial^2}{\partial x_i \partial x_j}\left[\nu_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\right] + \frac{\nabla_i U_i^n}{\Delta t} \tag{7}$$

The Reynolds number is chosen to be Re=22,000 so as to be consistent with the order of the Reynolds number in the compared experiments. The number of grid points is 320 × 200=64,000.

RESULTS

Identification of unsteady aerodynamic force coefficients

As well known, elongated cross-section produces motion induced forces by the interaction between the heaving and torsional motion and so-called coupled flutter is induced. Since it is a divergent vibration, which magnifies its amplitude as the increase of wind speed, one has to ascertain that its onset velocity is not within the region of design wind speed. For this purpose, flutter analysis is performed. For the two-degrees-of-freedom system which consists of heaving and torsion, aerodynamic lift and pitching moment are expressed as follows,

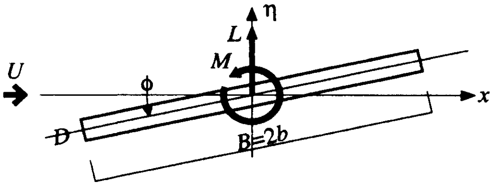


Fig.1. Definition of motion and force components

$$L=\frac{1}{2}\rho(2b)U^2\left(KH_1^*\frac{\dot{\eta}}{U}+KH_2^*\frac{b\dot{\phi}}{U}+K^2H_3^*\phi+K^2H_4^*\frac{\eta}{b}\right) \tag{8}$$

$$M=\frac{1}{2}\rho(2b^2)U^2\left(KA_1^*\frac{\dot{\eta}}{U}+KA_2^*\frac{b\dot{\phi}}{U}+K^2A_3^*\phi+K^2A_4^*\frac{\eta}{b}\right) \tag{9}$$

where $K=b\omega/U$ is reduced circular frequency, H_i^* and $A_i^*(i=1,4)$ are unsteady wind force coefficients and $b=B/2$ is a half chord length B. In the following calculations amplitudes is y_0/B=0.025 for heaving and ϕ_0=2 deg. for torsional motion, which followed the experiment by Matsumoto et al.(1994).

Unsteady aerodynamic force coefficients

In Fig.2 comparisons were made between the experimental and present numerical results. A coefficient which is often used for an index of torsional instability is A_2^*. $A_2^*>0$ implies possible occurrence of the one-degree-of-freedom torsional instability, i.e., torsional flutter, on the other

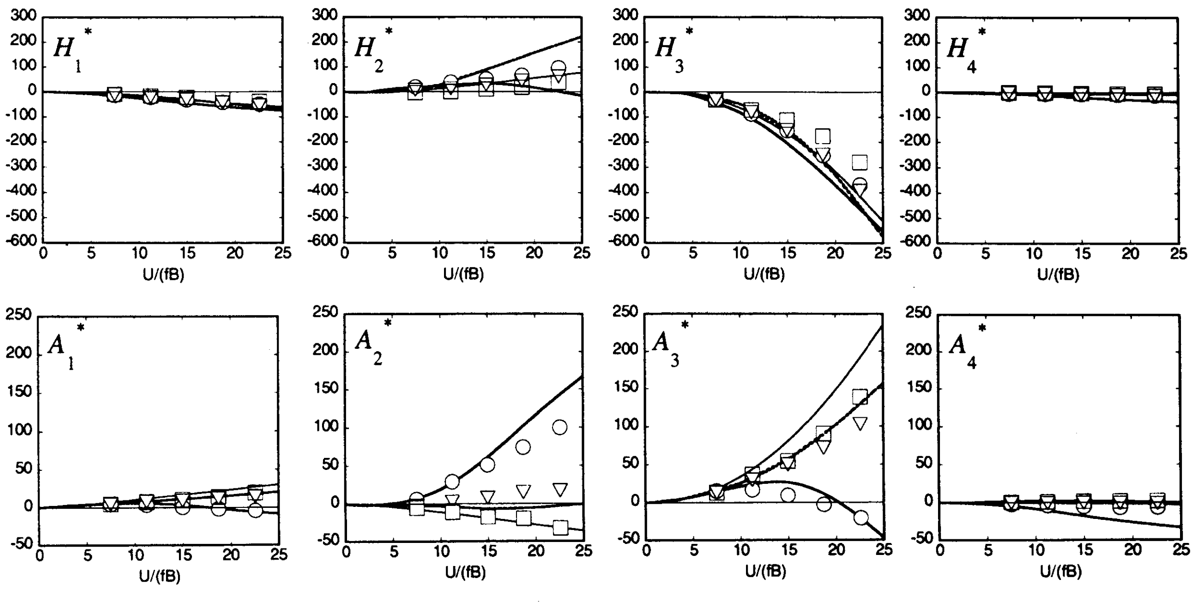
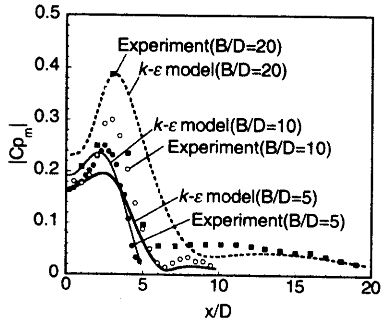
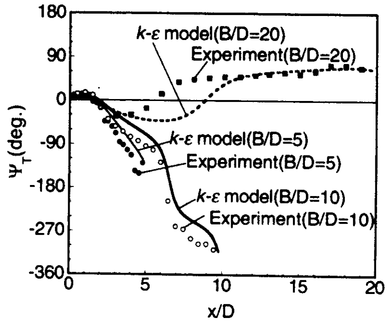


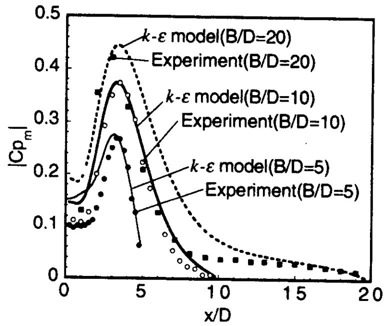
Fig.2. Aerodynamic coefficients of various rectangular cross-sections. Experiments were conducted by Matsumoto (1994) : ○; B/D=5, ▽ ; B/D=10, □; B/D=20, *k-ε* model : ——; B/D=5,—— ; B/D=10, —— ; B/D=20



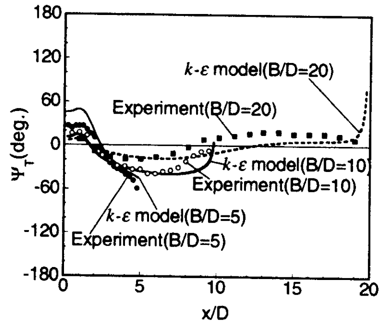
(a) Fluctuated pressure coefficients(U/fB=7.5)



(b) Phase differences(U/fB=7.5)



(c) Fluctuated pressure coefficients(U/fB=22.4)



(d) Phase differences(U/fB=22.4)

Fig.3. Comparison of unsteady pressure distribution obtained by the experiment by Matsumoto(1994) and the present method. (Torsional motion : ϕ_0=2 degree)

hand A_2^*<0 implies possible occurrence of the coupled-flutter. Experimental results which are represented by plots, A_2^* is positive for B/D=5, almost zero for B/D=10 and negative for B/D=20. These global tendencies are also well simulated by the present numerical method. These good agreement suggests the possibility of the present method on the use in the flutter analysis. In some points in detail, however, for example as can be seen in A_3^*, some inconsistencies also still remain in B/D=20. These points pose a necessity of further investigation.

Unsteady wind pressure distributions

Each unsteady wind force coefficient is an integrated value of the following unsteady pressure around a cross-section, which is defined as,

$$|Cp_m| = \sqrt{C_{P_R}^2 + C_{P_I}^2}, \quad \psi_T = \tan^{-1}\frac{C_{P_I}}{C_{P_R}} \tag{10a}$$

$$C_{P_R} = \frac{\frac{2}{T}\int_0^T p(t)\cdot \sin\omega t\, dt}{\frac{1}{2}\rho U^2}, \quad C_{P_I} = \frac{\frac{2}{T}\int_0^T p(t)\cdot \cos\omega t\, dt}{\frac{1}{2}\rho U^2} \tag{10b}$$

where, $|Cp_m|$ and Ψ_T is the fluctuated pressure coefficient and the phase difference with respect to the motion in frequency ω. In fig.3 comparisons are made for reduced wind speeds of $U/(fB)$=7.5 and 22.4. In the experiment which is presented by plots, each $|Cp_m|$ has a peak near x/D=3.5. From

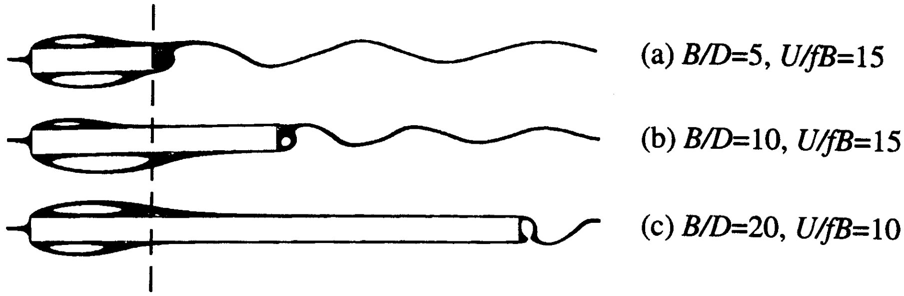
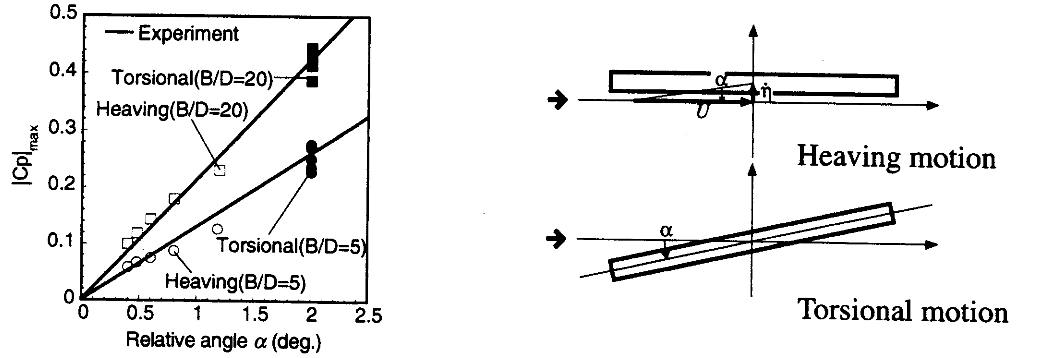


Fig.3. Variation of instantaneous streamlines. (Torsional motion: ϕ_0=2 degree. Vertical line is 5D from the left corner.)



(a) Maximum values of $|Cp_m|$ (b) Definition of the relative angle of attack

Fig.5. Relations between the relative angle of attack and the peak of the fluctuating pressure coefficients.Experiment is by Matsumoto

fig.4 the portion where $|Cp_m|$ is large corresponds, to the inside of the separation bubble. It can be recognized from fig.3 that the maximum size of the bubble L_B is approximately L_B/D=5. These are independent of the B/D ratio. This numerical result agrees well with what Matsumoto(1994) has pointed out that the reattachment of the separated shear layer is independent of the length of the after body. Phase difference seems to be expressed by a unique line for each reduced wind speed except for B/D=20 at $U/(fB)$=7.5.

Present numerical method successfully simulated these complicated nature of the unsteady pressure distribution for B/D=5 and 10. However, for B/D=20, slight differences are recognized in both of $|Cp_m|$ and Ψ_T near $5<x/D<9$.

In fig.5(a), peak value of $|Cp_m|$ is illustrated against the relative angle of attack, which is defined in the fig.5(b). As Matsumoto(1994) has pointed out, the linear relationship can be also recognized between $|Cp_m|$ and the relative angle of attack in the present numerical results.

Flutter characteristics

According to the method which was proposed by Miyata(1989), complex eigenvalues of the system which implements aerodynamic lift and pitching moment expressed as Eq.(8) and (9) are derived from the following equation.

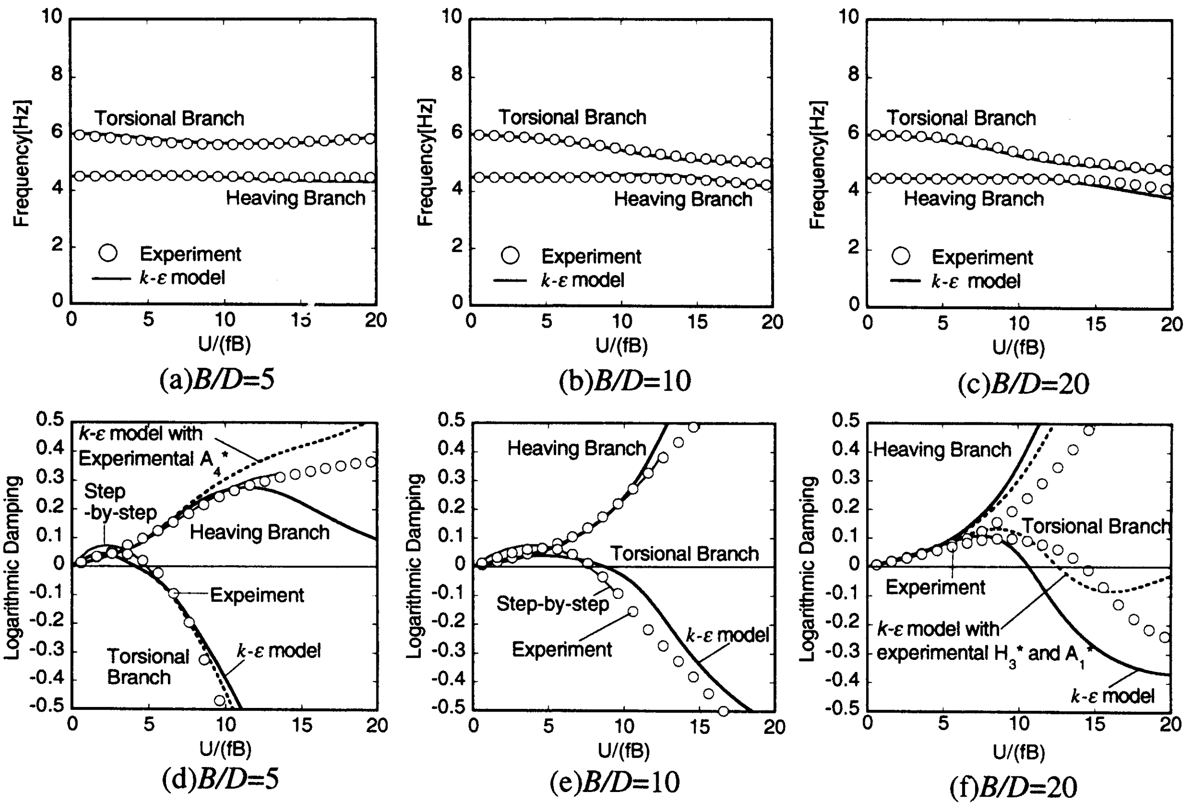


Fig.6. Eigenvalue loci of B/D=5, 10 and 20 rectangular cross-sections. M=1.96kg/m, I=4.9 × 10^3kgm, $f_{\eta 0}$=4.5Hz, $f_{\phi 0}$=6.0Hz, $B(=2b)$=0.15m

$$\omega^2 \begin{bmatrix} 1+\frac{\rho b^2}{M}(H_4^* + iH_1^*) & \frac{\rho b^3}{M}(H_3^* + iH_2^*) \\ \frac{\rho b^3}{I}(A_4^* + iA_1^*) & 1+\frac{\rho b^4}{I}(A_3^* + iA_2^*) \end{bmatrix} \begin{Bmatrix} H \\ \Phi \end{Bmatrix} = \begin{bmatrix} \omega_{\eta_0}^2 & 0 \\ 0 & \omega_{\phi_0}^2 \end{bmatrix} \begin{Bmatrix} H \\ \Phi \end{Bmatrix} \tag{11}$$

where, M and I are mass and mass moment of inertia per unit length, $\omega_{\eta 0}$ and $\omega_{\phi 0}$ are heaving and torsional circular frequency at rest respectively. This method is attractive since the complex eigen values can be readily obtained without any iteration. Circular frequency and logarithmic damping are determined from complex frequency $\omega_j = \omega_j^R + i\omega_j^I$ as follows,

$$\omega_F = |\omega_j|, \qquad \delta_j = 2\pi \frac{\Im(\omega_j)}{|\omega_j|} \tag{12}$$

In fig.6 eigenvalue loci of B/D=5, 10 and 20 which are representative of low-speed-torsional, high-speed-torsional and torsional-branch-coupled flutter type respectively are exhibited. Also in the figure, in order to validate the present eigenvalue analysis, result of the step-by-step analysis by Matsumoto(2001) are also exhibited. With respect to frequency, for each type the result by k-ε model are well agreed with the experimental result. Especially in fig.6(c), characteristic of torsional-branch-coupled flutter type are clearly recognized in the present simulated result, i.e., as the reduced wind speed becomes larger, both of the branches are getting closer each other. In the experiments, logarithmic damping of the heaving-branch is always positive and this characteristics are the same for the computational results. In the case of B/D=5, the experimental locus gradually increases, whereas in the computation it turns to decrease almost at fB/U=12. In order to clarify the reason of this difference, computational results in which only A_4^* is replaced by the experimental aerodynamic data are presented in the same figure. Since this difference was improved, the analytical precision in the prediction of A_4^* is found to be involved with. On the other hand, in the torsional-branch, the tendency is the same between the experiment and the computation, i.e. from a certain reduced wind speed the sign of the damping turns to be negative. In each B/D ratio, as the B/D ratio becomes larger the flutter speed also increases, however, in the case of fig.6(f), the present computation is conservative comparing with the experimental result. Result of "step-by-step analysis" by Matsumoto has suggested that A_2^* and production of A_1^* and $|H_3^*|$ are predominant in the contribution to the torsional-branch logarithmic damping. In fig.6(f), the computational result in which A_1^* and H_3^* were replaced by the experimental aerodynamic data also demonstrated the prediction was improved. Therefore the precision in the prediction of A_1^* and H_3^* is seemed to be concerned with and may be to the difference in prediction of the unsteady pressure distribution.

CONCLUDING REMARKS

Flutter characteristics of rectangular cylinders for B/D ratios of 5, 10 and 20 were calculated by means of two-dimensional analysis based on the k-ε model. The method was concluded to be effective for estimating global characteristics of the unsteady aerodynamic forces and flutter characteristic for B/D=5 and 10, however, for B/D=20 cross-section the estimation of flutter speed becomes conservative.

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